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AN AUTOMATIC WEATHER STATION

By Harry Diamond and Wilbur S. Hinman, Jr.

ABSTRACT

An important contribution of radio to the collection of weather data consists in the possibility of utilizing completely automatic stations installed at isolated locations, such as mountain peaks, islands, etc., whereby information on the meteorological factors of interest is automatically transmitted at predetermined intervals. Details are given of a method for the remote indication by radio of measurements of barometric pressure, air temperature, relative humidity, wind direction and velocity, rainfall, and other meteorological factors. The method was developed to meet requirements formulated by the Aerological Division, Bureau of Aeronautics, Navy Department, the work being done for that Service.

The method retains the operating principles of the audio-modulation type radio sonde but, because of the use of much lower audio frequencies (0.15 to 3.0 cycles per second), affords important simplifications in the transmitting and receiving equipment. The low frequencies allow mechanical keying of a conventional radio telegraph transmitter, thereby avoiding the need for complicated modulating equipment. At the receiving end, the frequency measuring and recording functions are carried out by simple electric counters or by manual counting, which eliminate the expensive electronic-frequency meter and recording microammeter used at radio-sonde ground stations.

An experimental installation set up at the Naval Air Station, Anacostia, D. C., using this method, is described and measurements for a 30-day period are analyzed.

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I. INTRODUCTION

The development of methods for the remote indication of meteorological measuring instruments has received the attention of experimenters for many years. In recent years, experimental work has been focused on modifying the methods already devised for wire lines to accommodate their use with radio and on the development of methods more adaptable to radio technique. Examples of important applications of radio telemetering in meteorology are: the radio sonde, the radio river-stage recorder, and the automatic weather station. The radio sonde on the one hand, and the other two applications on the other hand, present sharply different operating problems. It is of interest to consider these differences briefly from the point of view of cost and performance requirements.

In the radio-sonde application, it is important that the cost of the balloon instrument be kept small since it is used but once, but considerable leeway is possible in the cost of the ground-station receiving and recording equipment. Here the emphasis is on performance and speed of operation rather than on cost. The justification lies in the need for rapid evaluation of a large number of observations so that the observed data may be available in time for making scheduled forecasts.

The radio river-stage recorder and the automatic weather station differ from the radio sonde in these respects. The major cost of the system is represented in the transmitting installation, because this end of the system must be capable of unattended operation for periods up to 3 months and the required accuracy of measurement is somewhat greater than for the radio sonde. Since only a few measurements are made during each operating sequence, the need for low observation time and rapid evaluation at the receiving end is reduced, so that, in some instances, manual observations are sufficient. This renders possible a material decrease in the cost of the receiving station. The change in the ratio of cost of the transmitting to the receiving station (as compared to the radio sonde) is further accentuated by the nature of the radio path. The service may be over a great distance, so that reliance must usually be placed on ionospheric propagation. The efficiency of propagation is substantially lower than for the line-of-sight path available in radio-sonde operation, and there is a consequent increase in the transmitter power required. Moreover, for reliable 24-hour operation, more than one operating frequency may be required.

An important difference in operating requirements of the radio sonde and the other two applications lies in the speed of response of the measuring elements. The radio sonde requires high rates of response so that the time for a complete sounding may be kept within practicable limits. The other two applications can tolerate greater lag in response, since such lag merely introduces temporal displacements in the observations rather than the altitude displacements for the radio sonde.

So far, it has been shown that the radio river-stage recorder and the automatic weather station are quite similar in many respects. One difference arises in the design of means for converting the deflections of the measuring devices into characteristics of the emitted radio wave which may be interpreted at the receiving end. The float and pulley

of the river-stage instrument provide ample power for operating practically any keying or coding mechanism which can be devised, whereas the delicate instruments of a weather station afford very little direct power for such use. This difference justifies the use of substantially different telemetering methods for the two applications [1, 2].[1]

II. THE METHOD OF REMOTE INDICATION

In an automatic weather station, measurements may be required of barometric pressure, ambient temperature and relative humidity, wind velocity and direction, rainfall, visibility, ceiling height, and probably other factors. It is evident that each of the measurements noted may be indicated at a remote point by radio by converting the deflection of the measuring instrument into a corresponding variation of electrical resistance and translating the resistance variation into the variation of a modulation frequency on the emitted carrier wave. This is the method used in the audio-modulation type radio sonde [3].

1. THE MODULATING SYSTEM

The translation of resistance variations into corresponding variations of modulating frequency is accomplished by means of a trigger-type modulating oscillator. The circuit arrangement is shown in figure 1.

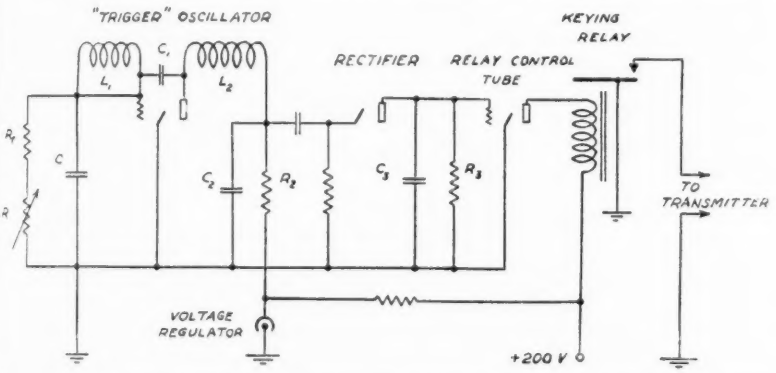

FIGURE 1.—*Electric-circuit arrangement of system for keying radio transmitter.*

The oscillatory circuit, $L_1C_1L_2$, is tuned to a frequency of about 1 Mc/s. The 1-Mc oscillations are interrupted at an audio rate determined by the time constant of the resistance-capacitance combination, R_fRC. Here R is the variable resistor (generally in series with a limiting reference resistor, R_f, of about 40,000 ohms) and C is a stable condenser of about 8 μf capacitance. For the range of resistance values used in the automatic weather station, the range of modulation frequencies extends from about 0.15 to about 3.0 c/s. Because of the low frequencies used, it is convenient to speak of them as so many pulses per second.

The output of the modulating oscillator is coupled to a half-wave rectifier which, in turn, applies negative voltage to the input of the

[1] Figures in brackets refer to the literature references at the end of this paper.

relay control tube. The smoothing filter, R_2C_2, in the plate circuit of the modulating oscillator, converts the groups of 1-Mc oscillations into corresponding audio-frequency pulses. After rectification, these appear as negative pulses on the grid of the relay control tube; their duration is controlled by the time constant of the network, R_3C_3. The relay is normally in the actuated position; the armature is released upon application of the negative pulses to the grid of the relay control tube and remains open so long as the grid bias is sufficient to keep the current through the relay coil below the "pull-in" value. The back contact of the relay may thus be used for keying any conventional radiotelegraph transmitter; there is seen to be no limitation on the power output of the transmitter.

The use of low modulation frequencies (0.15 to 3.0 pulses per second) not only provides for simple modulation of a radio transmitter but also affords considerable simplification in the equipment for measuring and recording the modulation frequencies at the receiving station. Manual observations may be made simply by counting the number of pulses received in a given time interval (20 to 40 seconds). A stop watch and headphones are sufficient for the purpose. Recorded observations may be obtained with a simple tape recorder which provides distinguishable tenth counts and a timing mark on the tape.

As in the radio sonde, the resistor, R, in the grid circuit of the modulating oscillator may be varied mechanically by the instrument deflection, or resistors which vary inherently in response to the factors measured may be used. In the experimental setup to be described, the first arrangement is used in the instruments for measuring pressure, humidity, rainfall, and wind direction, whereas the second arrangement is applied to the measurement of temperature and will probably be used in the measurement of visibility and ceiling height. Wind velocity is measured by the direct conversion of the rotation of the cup anemometer into a variable audio frequency.

2. RESISTORS VARIED MECHANICALLY BY MEASURING INSTRUMENTS

Figure 2 shows the mechanism used for converting the deflection of the barometer into a resistance variation. The instrument pointer, *a*, swings freely between a clamping bar, *b*, and the edge of a 100,000-ohm wire-wound resistor, *c*. The clamping bar moves about the pivot, *d*, and serves to clamp the pointer against the resistor when the relay, *e*, is actuated. The variable resistance is formed between one end of the resistor and the instrument pointer which constitutes the variable contact point. The field coil of the relay is excited when an observation of pressure is required. (The timing and switching arrangement for starting the automatic weather station and for selecting the observations from the several instruments in the desired sequence will be described later in the paper.) Details of the linkage system for operating the pointer, *a*, from the evacuated bellows will be evident from a study of the two views of figure 2.

It will be evident that this general scheme for converting the deflection of an instrument pointer into a proportional value of resistance may be applied to any instrument no matter how delicate. An obvious advantage lies in the fact that the pointer reaches its reading position without friction introduced by the electric contact.

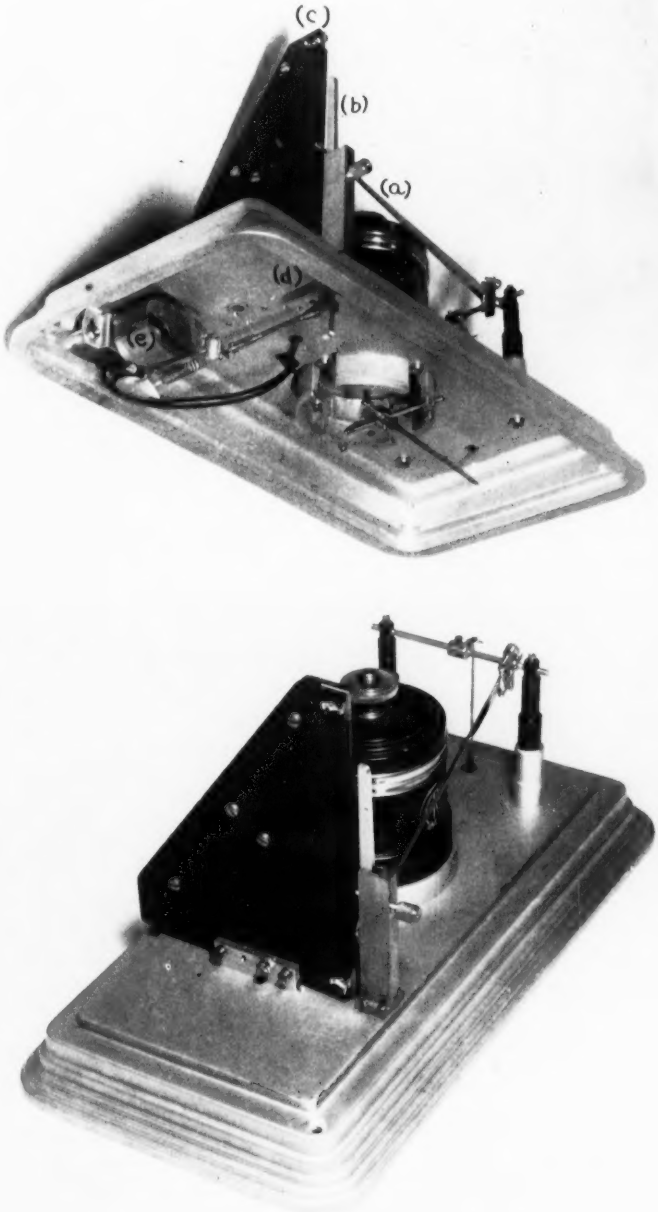

FIGURE 2.—*Mechanical arrangement for converting deflection of barometer into a resistance variation.*

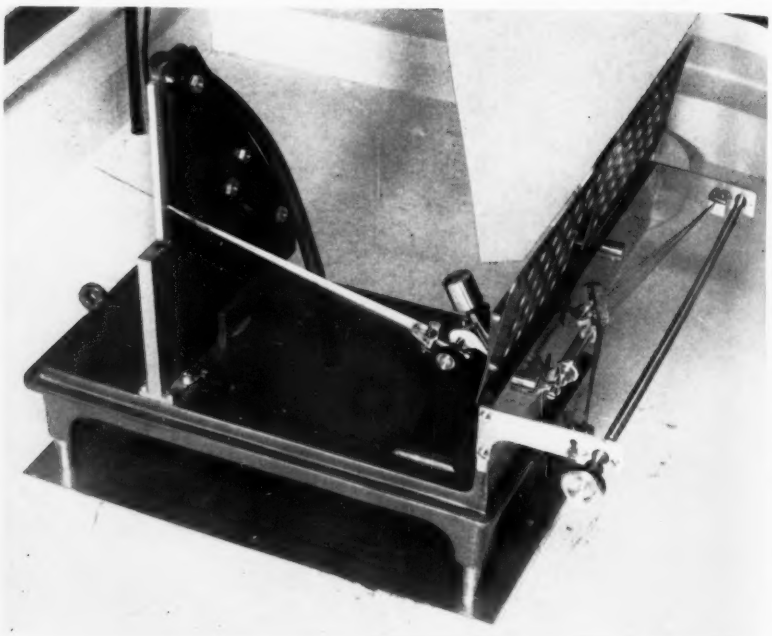

FIGURE 3.—*Hair hygrometer controlling a variable resistor.*

FIGURE 4.— *Weighing-type rain gage controlling a variable resistor.*

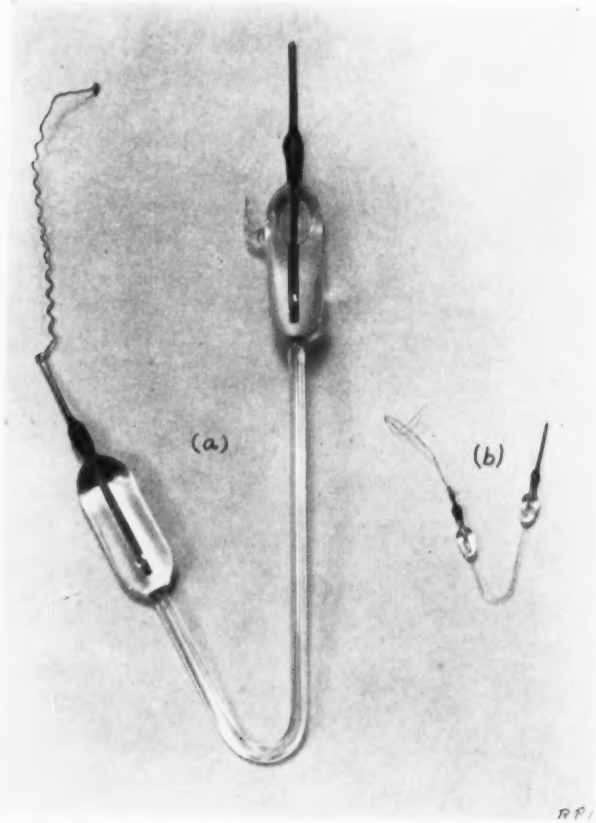

FIGURE 5.—*Inherently variable resistor responsive to ambient temperature.*
a, Unit used in automatic weather station; *b*, unit used in audio-modulation type radio sonde.

The application of this scheme to the hair hygrometer and to the weighing-type rain gage will be evident from figures 3 and 4, respectively.

The conversion of the position of a wind vane into a corresponding value of resistance is most readily accomplished through the use of a series of different fixed resistors which are switched into circuit as the wind vane passes through its principal positions. The standard wind vane used in the experimental setup has eight equally spaced switching positions; hence, eight different resistors are used. From this it would appear that only wind directions corresponding to the eight principal compass points are indicated. However, it is possible to provide effective indication for eight additional points intermediate to the principal points by taking advantage of the fact that the wind vane oscillates considerably about its average position when the wind velocity exceeds a few knots per hour. The cams for switching in the resistors cover only an arc of just under 45 degrees (instead of 60 degrees in the standard wind vane), so that if the wind direction corresponds to a principal position only one resistor is connected into circuit. However, if the wind direction is intermediate to two principal directions, the two corresponding resistors will be switched successively into circuit by the oscillation of the vane, so that the number of pulses counted in a convenient time interval (say, 15 seconds) will represent an average value and thus indicate an intermediate direction.

3. INHERENTLY VARIABLE RESISTORS

The inherently variable resistor used for measuring air temperature is shown in figure 5, at *a*. It is similar in all respects except size to the electrolytic thermometer, *b*, used in the audio-modulation type radio sonde. It consists of a glass capillary tube filled with an electrolyte having a high temperature coefficient of electrical resistance and provided with terminals so that it may be used as a resistor whose value varies in accordance with the ambient temperature. The capillary bore is only 0.75 mm in diameter; the wall thickness (although contributing to lag in response) adds materially to the mechanical sturdiness of the device. The large end wells and heavy copper terminals provide for almost indefinite life of the device under the conditions of intermittent service contemplated for the automatic weather station. For the length of capillary shown (22 cm), the device has a resistance of 40,000 ohms at +40° C; at −40° C. its resistance increases to 480,000 ohms. The time-lag constant [4] of the device in gently moving air (25 m/min) is 2 min compared to 20 sec for the radio-sonde tube with the same ventilation. As previously indicated, the increased lag in response is of little importance; temperature changes of the order of 0.5 degree per minute are not frequent, and even in such cases the error introduced would be only 1 degree.

It is evident that the use of the electric hygrometer [5] for measuring relative humidity is feasible. An experimental unit was incorporated in the setup to be described, but final design details have not been completed.

Graph *a* of figure 6 shows the variation of modulation frequency with resistance for the modulating oscillator of figure 1. The abscis-

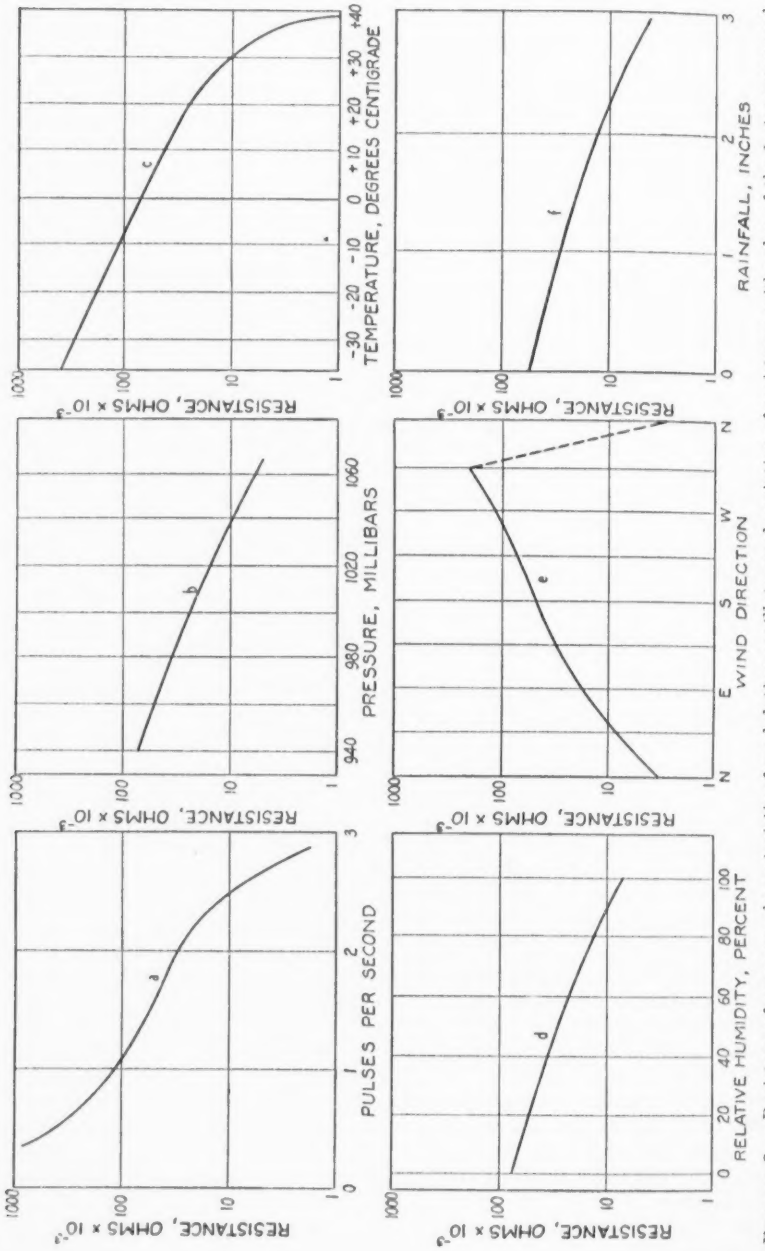

FIGURE 6.—*Resistance-frequency characteristic of modulating oscillator and variation of resistance with value of the factor measured for the several measuring instruments.*

(Add 40,000 ohms to the ordinate scale for graph *c* to obtain the true value of resistance corresponding to a given temperature.)

sas represent the generated audio frequencies in pulses per second while the ordinates correspond to the values of the resistance R. Graphs, *b*, *c*, *d*, *e*, and *f* of figure 6 show the range of variation of resistance obtained corresponding to the indicating ranges of the

pressure, temperature, humidity, wind direction, and rainfall instruments respectively.

When any one of the several instrument resistors is connected as R of figure 1, the variation of the pulses per second as a function of the meteorological factor being measured is as shown by the graphs of

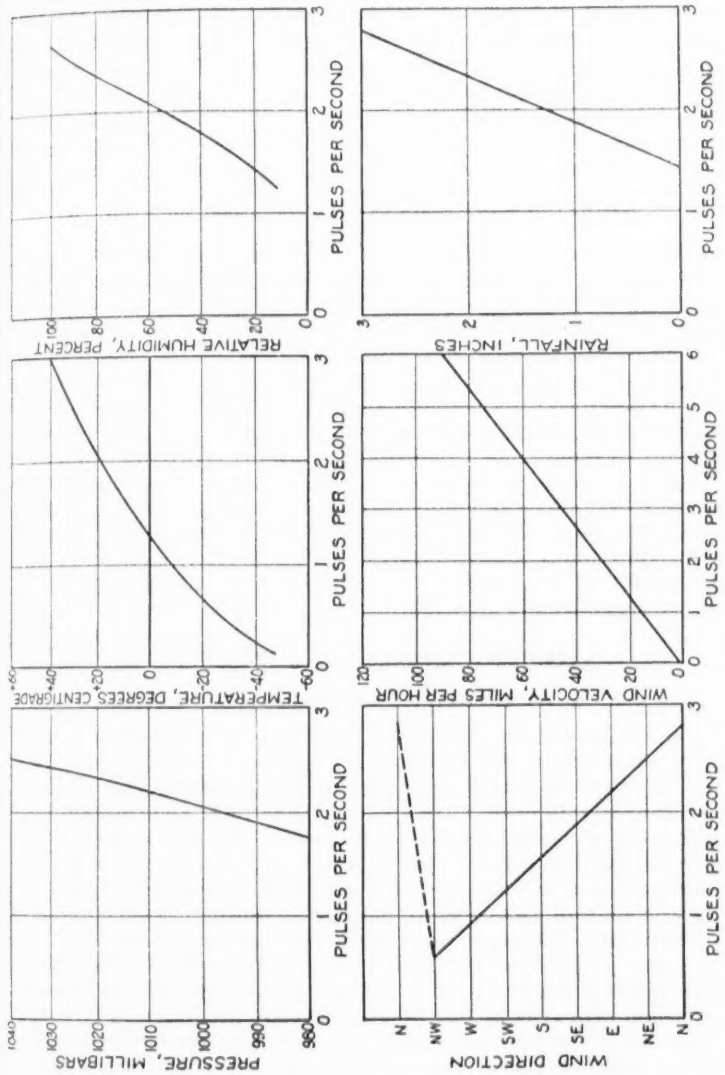

FIGURE 7.—*Calibration curves for measuring instruments showing indicating frequency range for each of the factors measured.*

figure 7. These are based on a reference frequency of 3.0 pulses per second (that is, when $R=0$ and only the reference resistor of 40,000 ohms remains in circuit). When the reference frequency deviates from exactly 3.0 pulses per second, the frequencies corresponding to various values of R may be corrected proportionally. Hence, the

graphs of figure 7 constitute a calibration for the automatic weather station.

For the temperature tube, connection of the device in series with the reference resistor would result in a value of resistance at +40° C of 80,000 ohms and a corresponding upper-frequency limit of 1.7 pulses per second. Since this would unduly restrict the indicating range, the reference resistor is automatically disconnected when the temperature tube is connected into circuit. The upper temperature-indicating frequency then becomes 3.0 pulses per second. The reason for connecting the other instrument resistors in series with the reference resistor is that the modulating oscillator becomes unstable for values of resistance below about 40,000 ohms and certain of the instrument resistors drop below this value in certain portions of their range.

4. DIRECT KEYING BY CUP ANEMOMETER

As previously noted, the rotation of the cup anemometer may be converted directly into a modulation frequency proportional to wind velocity, without requiring an intermediate conversion into a variable resistance. This is accomplished by stepping up by a factor of four the number of "take-off" contacts in a commercial cup anemometer; one contact then occurs for each three revolutions of the cups. The cup arm is chosen so that the number of contacts occurring in a 15-sec interval is numerically equal to the wind velocity in knots per hour. The contact terminals are connected in parallel with the contacts of the keying relay and pulse the radio transmitter in the same way as the keying relay. The range of indication is seen to be from 0.06 to 6.0 pulses per second for wind velocities ranging from 0.9 to 90 knots per hour. The modulation frequency corresponding to a given wind velocity is seen to have an absolute value and is not subject to correction for reference-frequency drift.

III. THE CONTROL SYSTEM

1. AUTOMATIC ROTARY SWITCH

The control system for automatic operation of the weather station is shown diagrammatically in figure 8. The heart of the system is a rotary step relay or switch of standard design, having two parallel banks of 25 contacts each. This switch incorporates a ratchet relay and a grounded moving arm, *A*. The pawl is set up for action when the relay field coil is excited and advances the moving arm one step when the excitation is removed. (In the illustration, the arm is shown bridging the pair of switch contacts corresponding to positions 1 in the two banks). The remaining contacts of the inner bank, 2 to 25 inclusive, are connected to the field coil of the control relay; hence, this relay is excited by a 12-volt battery (with negative ground connection) when arm *A* is in any position except the first. The control relay operates a double-pole armature; one pole controls the power relay of the radio transmitter and the second pole controls the 12-volt power circuit to a small, constant-speed motor.

(a) HOMING ACTION

Contacts 19 to 25, inclusive, of the outer bank of the rotary switch are connected to the armature of the ratchet relay; the armature normally rests against the fixed contact of the ratchet relay, moving

away from it when the relay is actuated. The net result is to secure "homing" action of the rotary switch once the grounded arm, *A*, reaches position 19, as will be evident from a study of the connections.

(b) CODING CONTACTS

The even-numbered contacts, 4 to 16, inclusive, of the outer bank of the rotary switch are connected to a set of coding circuits which provide the station identification signals and the coded characters *P*, *T*, *H*, *D*, *V*, and *R*, corresponding, respectively to pressure. temperature, humidity, wind direction, wind velocity, and rainfall,

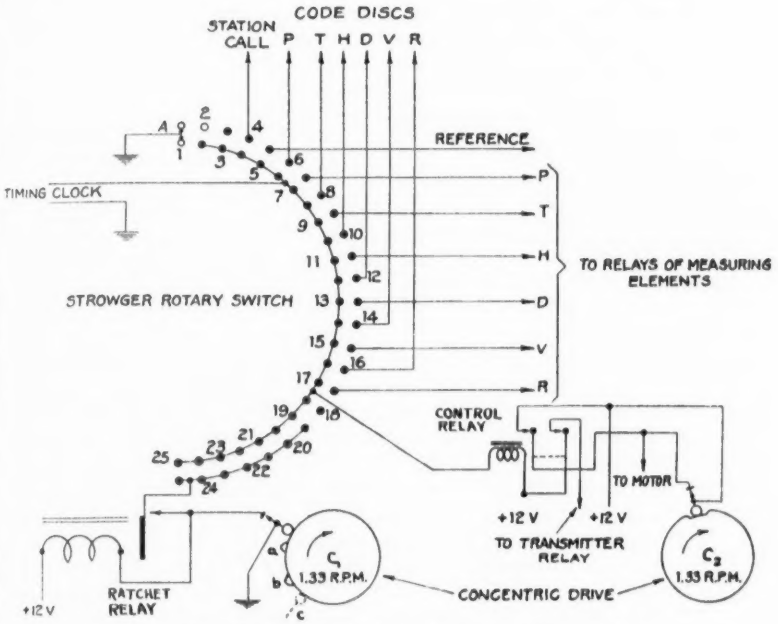

FIGURE 8.—*Control system for automatic weather station.*

Other code letters corresponding to visibility, ceiling, height, etc., may be added by adding the even-numbered contacts 18, 20, etc., to the coding network. The coding circuits consist of cam-operated switches which are connected in parallel with the telegraph key of the radio transmitter through the grounding of one side by the appropriate even-numbered contacts of the rotary switch.

(C) OBSERVATION CONTACTS

The odd-numbered contacts 5 to 17, inclusive, of the outer bank of the rotary switch are connected to relays associated with the measuring elements for connecting the variable resistors, *R*, in the modulating circuit (see fig. 1) and for operating the clamping bars of the pressure, humidity, and rain gage instruments (see figs. 2, 3, and 4). Other measuring-element resistors, such as photoelectric cells for

measuring visibility, ceiling height, etc., may be added by using contact numbers 19, 21, etc.

Contact 18 of the outer bank is left blank. The reason for this will appear from the following paragraphs.

2. OPERATION OF SYSTEM

The operation of the control system may now be outlined. At preset times of day, a clock-operated switch grounds the inner-bank contacts 2 to 25 of the rotary switch and keeps them grounded for approximately 10 min. This excites the field coil of the control relay and operates the double-pole armature. The radio transmitter is thus started and, also, the small 12-volt motor. The latter drives two cams, C_1 and C_2, on a common shaft making a complete revolution once each 45 sec.

(a) SEQUENCE CAM

Cam C_1 has two raised segments, a and b, 5 sec apart from a to b and 40 sec apart from b to a. These operate an associated cam switch which completes the battery circuit through the field coil of the ratchet relay and advances the arm of the rotary switch one step for each connection. Assuming that the raised segments are in the positions shown in figure 8 at the beginning of operation, the sequence of operations will be as follows:

Segment a of cam C_1 closes the cam switch, so that the field coil of the ratchet relay is excited and the armature is pulled over. When segment a moves on, the cam switch opens releasing the armature and operating the ratchet on the rotary switch so that the moving arm A advances to step 2. It will remain on step 2 for 5 sec, when segment b on cam C_1 again operates the system to advance arm A to step 3, where it remains for 40 sec—that is, until segment a passes by the cam switch again. It will thus be evident that the moving arm A will advance steadily, remaining on the odd-numbered contacts for intervals of 40 sec each and on the even-numbered contacts for intervals of 5 sec each. Contacts 2 and 3 allow for warm-up period for the radio transmitter. The remaining contacts provide for the connections of the coding circuits and instrument resistors, as already indicated. The complete sequence of signals from the automatic weather station is shown in table 1. At the end of the observation period corresponding to contact 17, segment a comes by the cam switch, thereby moving arm A to contact 18. This contact is left blank to allow segment b to pass the cam switch before homing of the rotary switch occurs, so as to leave cam C_1 in proper position for starting a second sequence of signals.

The homing feature of the rotary switch brings the moving arm to its zero position (contact 1) after the completion of a complete sequence of operation. However, since a complete sequence takes about 6 min and since the clock-switch remains closed for about 10 min, a second sequence of operations automatically begins. Once switch arm A has moved onto contacts 2, the completion of the second sequence is assured, since, even if the clock-switch opens, the field coil of the control relay remains excited until the rotary switch returns to its zero position.

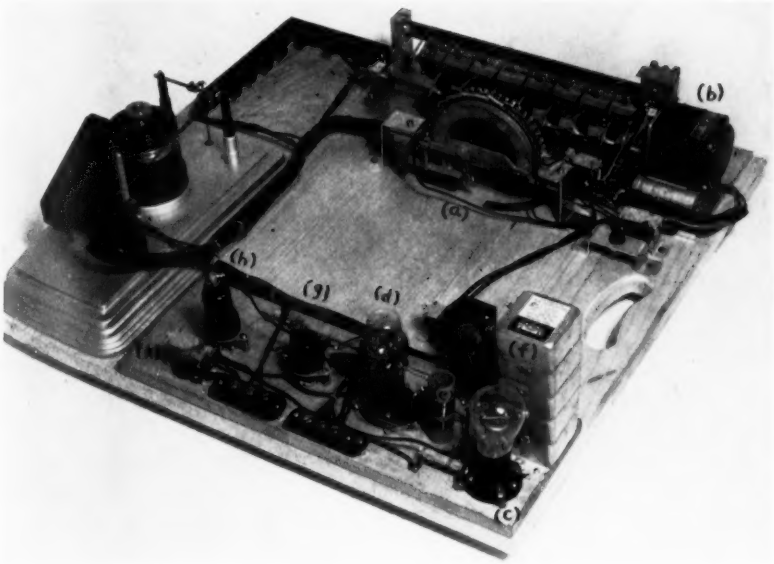

FIGURE 9.—*Control and modulating equipment.*

TABLE 1.—*Sequence of signals from automatic weather station.* [a]

Contact Number	Function	Time of duration
		Seconds
2 and 3	Transmitter warm-up	45
4	Station call letters	5
5	Reference frequency	40
6	Code letter P (. — — .)	5
7	Pressure frequency	40
8	Code letter T (—)	5
9	Temperature frequency	40
10	Code letter H (. . . .)	5
11	Humidity frequency	40
12	Code letter D (— . .)	5
13	Wind direction frequency	40
14	Code letter V (. . . —)	5
15	Wind velocity frequency	40
16	Code letter R (. — .)	5
17	Rainfall frequency	40
18	Blank	5
19 to 25	Homing of relay	2

[a] Sequence repeated twice during each operation.

(b) LIMIT-STOP CAM

The function of cam C_2 may now be considered. After the second sequence of signals has been completed and switch arm A has returned to zero position, it will be observed that cam C_1 will normally be in a position such that its switch rests at about c. The control relay has opened so that, were it not for the cam switch operated by the cam C_2, the motor would stop. However, this cam switch remains closed except when the depressed section of the cam passes under it. The motor is thus kept running until this point, so that cam C_1 is automatically brought to the position shown in figure 8 before the motor stops. The system is thus set up properly for the next operating period. The operation of cam C_2 is normally not important. However, it guards against the possibility of reversal of the time of duration of the code and observation signals in the event of momentary failure of the rotary switch—for example, missing one contact.

3. CONTROL AND MODULATING EQUIPMENT

Figure 9 shows the control and modulating equipment used. The control equipment is at the back and the modulating equipment at the front. (For convenience, the barometer is included in this assembly, at the left.) The equipment is seen to be relatively simple. In the control equipment, the rotary switch is shown at a, the motor at b, and the coding disks and cams C_1 and C_2 (with the associated switches) in the bank behind the rotary switch. The two single-pole relays shown to the right rear and right of the rotary switch are used as the control relay in figure 8. In the modulating equipment, c is a voltage-regulator tube to assure constant plate-voltage supply to the modulating oscillator, d, thereby reducing reference-frequency drift; voltage is obtained from the dynamotor which furnishes plate supply to the radio transmitter. The coils L_1 and L_2 used in the modulating oscillator are shown at e; and the condenser, C, of the time-constant circuit is shown at f. The rectifier tube and the relay control tube are shown, respectively, at g and h and the keying relay at k.

IV. THE COMPLETE STATION

1. ELECTRIC-CIRCUIT ARRANGEMENT

A schematic diagram of the electric connections of all the component parts of the station is shown in figure 10. The electric-circuit arrangement of the modulating equipment (shown in fig. 1) is included for completeness and need not be discussed again. It will be seen that the keying relay is connected in parallel with the conventional telegraph key of the radio transmitter and is essentially an automatic key. The various relays associated with the odd-numbered contacts of the rotary switch are given the same numbers as the contacts. Relay 5 connects only the 40,000-ohm reference resistor in the grid circuit of the modulating oscillator, thereby providing a reference frequency—nominally 3.0 pulses per second. This frequency serves as a check on frequency drift of the oscillator and as a means for correcting the various observations (except wind velocity) for frequency drift. Relays 7, 11, and 17 connect the corresponding instrument resistors in series with the reference resistor; companion relays operate the clamping bars for these instruments. Relay 9 connects in the temperature resistor directly, leaving the reference resistor out of circuit as already noted. The functions of relays 13 and 15 used with the wind-indicating instruments are self-evident. Note that the contact switch in the wind-velocity indicator is connected in parallel with the keying relay and keys the transmitter directly. The code keys (formed by the code disks and associated switches) are similarly connected, operating only when the moving arm of the rotary switch is on the even-numbered contacts. No details of the radio transmitter are given, since it may be any conventional radiotelegraph unit; a tone-modulated transmitter affords the possibility of using simple filters at the receiving station for increasing the signal-to-noise ratio.

2. EXPERIMENTAL EQUIPMENT

A photograph of an experimental automatic weather station installed at the Naval Air Station, Anacostia, D. C., is shown in figure 11. The unit has three separate compartments. The top compartment takes the form of the conventional ventilated shelter for housing the temperature- and humidity-measuring elements; it serves also as a support for the wind-measuring instruments. An inside view of this compartment is shown in figure 12. The middle compartment houses the control and modulating unit, already shown in figure 9. The lower compartment houses the radio transmitter, power supply, and timing clock.

3. CONSIDERATIONS OF RADIO TRANSMITTER POWER AND OPERATING FREQUENCIES

There are a number of practical considerations which influence the size of the radio transmitter and the radio frequencies to be used. Since the station is to be installed at isolated locations, the primary power supply is necessarily a storage battery. Although the storage

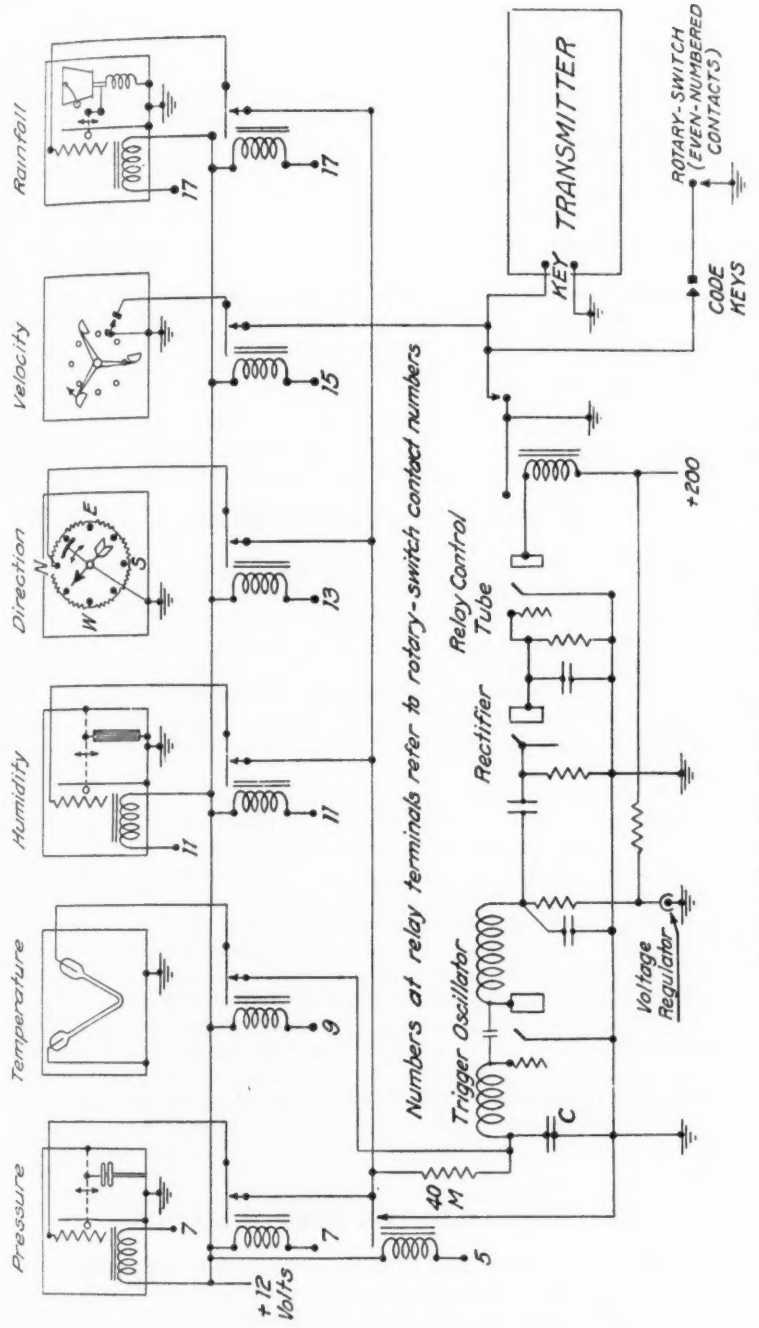

FIGURE 10.—*Circuit arrangement of automatic weather station.*

battery may generally be kept fairly well charged by a wind-driven charging generator, some provision must be made for operation during calm weather. A 15-watt transmitter, together with the control and modulating equipment described, requires an average input current of about 7.5 amperes at 12 volts. Each set of observations (two sequences) takes about 15 min., and observations may be required at hourly intervals. Hence, the daily ampere-hour load is 45 ampere-hours. A storage-battery capacity of about 200 ampere-hours appears reasonable for this service. It is evident that any large increase in transmitter power output would result in an impracticable size of battery.

The distance range of reception required in this service may be from 25 to 200 miles, and, except in unusual cases, a line-of-sight path is not available. Hence, propagation by way of the ionosphere is indicated, at least two radio frequencies (a day and a night frequency) being required. The frequencies of 5072.5 kc/s and 3135 kc/s are being used in the experimental setup at Anacostia. An arrangement for automatic change-over from day to night frequency and vice versa has not yet been incorporated but presents no difficulties.

V. THE RECEIVING SYSTEM

An ordinary receiving set may be used in the reception of signals from the automatic weather station. Where operating conditions allow, manual observations are preferable, since accurate observations may then be obtained through severe interference. For a given measurement, the operator measures the time for a given number of pulses (usually 30 to 70). Simple division resolves the indication to pulses per second. This value is corrected proportionally for reference-frequency drift, and the observation is evaluated by means of an appropriate chart.

Figure 13 shows a recorder designed for use with this type of signal when an automatic record is required. The details of the recorder are obvious. The tape is driven at constant speed by a synchronous motor. A time marker is provided on the tape at 1-sec. intervals by means of a cam-operated switch. The received pulses (in the output of the receiving set) are impressed on a printer relay which provides a mark for each pulse. A second relay, linked to the first, provides a distinguishing mark for each tenth pulse. It is thus quite easy to determine from the record the time required for the reception of a given number of pulses. More complex recorders, which automatically plot the values of the factors measured as a function of time, have been considered. However, such recorders do not appear to be warranted, partly because of the intermittent observations and partly because the signal-to-noise ratio under the conditions of service outlined is likely to prove too low in many instances to allow the use of a recorder.

VI. EXPERIMENTAL PERFORMANCE

To determine the serviceability of the automatic control equipment and the accuracy of the measurements obtained, the experimental automatic weather station described was installed on the roof of the Aerology Building, Naval Air Station, Anacostia, D. C., and operated

FIGURE 11.—*Experimental automatic weather station installed at Naval Air Station, Anacostia, D. C.*

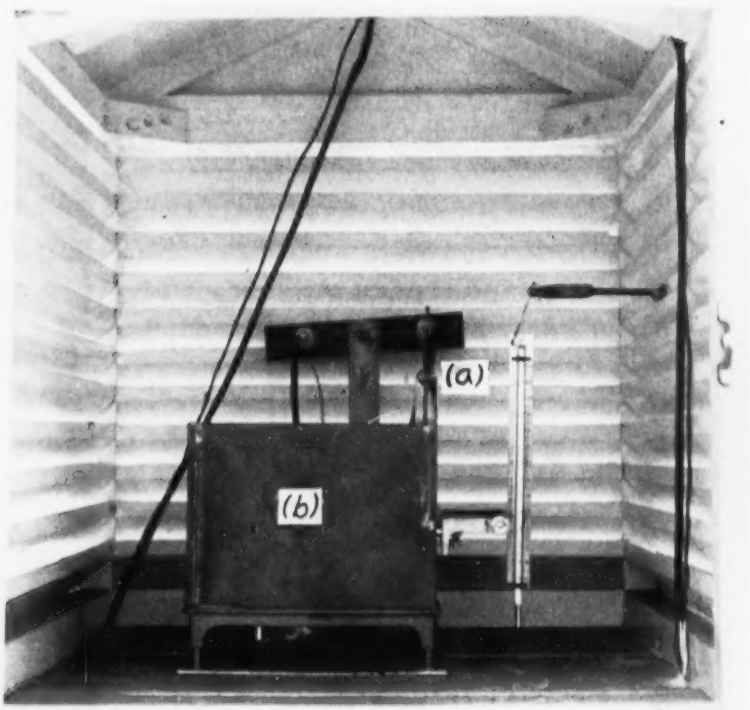

FIGURE 12.—*View inside ventilated shelter, showing temperature unit at (a) and humidity unit at (b).*

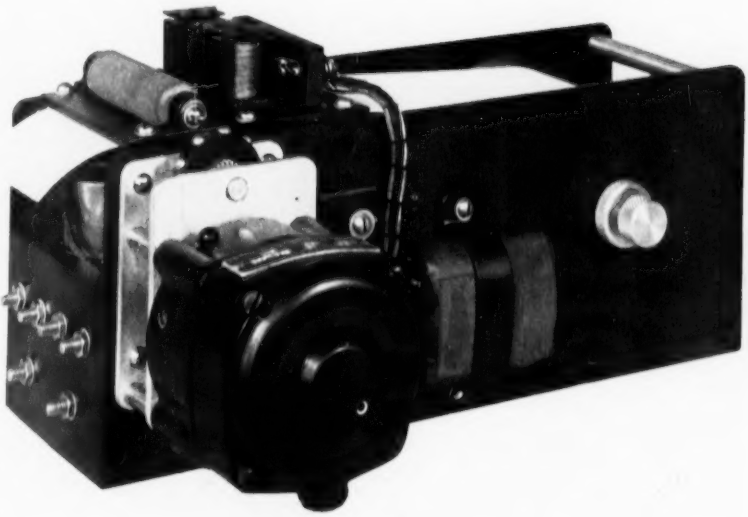

FIGURE 13. *Tape recorder.*

during April, 1940, three times daily. The operation was entirely automatic except for occasional additional sets of observations for experimental and demonstration purposes. Observations were made by radio at the Bureau laboratories and were compared with routine simultaneous visual observations at the Naval Air Station.

Several minor failures of the equipment occurred during the month. These were all of a mechanical nature and were almost wholly due to the experimental timing and coding mechanism. Satisfactory repairs were made on the spot. No failures were noted which could be attributed to the method or to the measuring elements used. Four different operators, using the manual observation method, took readings on this station and there was no noticeable difference in the accuracies obtained.

The accuracy of the measurements obtained is indicated in figure 14, which summarizes all the measurements for April. Results are

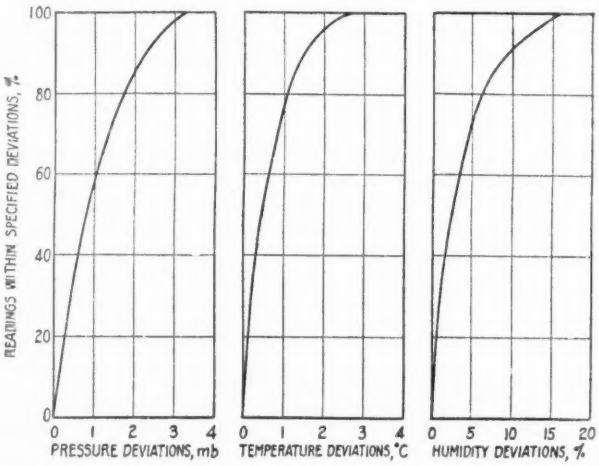

FIGURE 14.—*Summary of deviations of observed pressure, temperature and humidity measurements from values as measured with standard weather instruments.*

shown only for barometric pressure, temperature, and relative humidity; the error in wind direction was negligible, the error in wind velocity was less than 1 knot per hour, and the number of observations for rainfall proved insufficient for accurate evaluation of error. In figure 14, the abscissas represent deviations between the observed values and the routine measurements at the Naval Air Station with conventional weather instruments; the ordinates plot the percentages of the observed values which agreed within specified deviations. It will be seen that 80 percent of the pressure readings agreed within 1.75 millibars, 80 percent of the temperature readings within 1.1 degrees centigrade, and 80 percent of the humidity readings within 6 percent relative humidity. Some fraction of the errors must be attributed to the comparison instruments and some fraction to personal error; an exact evaluation is not yet available.

It is expected that some improvement in the pressure measurement may be obtained by the use of a barometer having a narrower indicating range or a wider resistance range. Either change would

result in a much greater range of frequency indication for the range of pressure variation used. (See figs. 6 and 7.) The accuracy of the temperature measurements is believed to be better than indicated in figure 14, a fixed calibration difference of 0.5 degree centigrade between the temperature tube and the comparison thermometer having recently been detected.

VII. CONCLUSION

In conclusion, it is evident that the system is quite simple. It allows the use of standard radio transmitting and receiving equipment and of standard weather instruments with but slight modification. The modulation and control equipment is special but is more complicated in explanation than in actual design or operation. As described, the system is readily adaptable to the addition of other measuring elements, such as the electric hydrometer for improved measurement of relative humidity, photoelectric cells for measuring visibility and related factors, etc. The signals may be interpreted without the need of special measuring and recording equipment. Use of the system may thus be made by field crews.

The authors wish to acknowledge their appreciation to Commanders W. M. Lockhart and J. B. Anderson, of the Bureau of Aeronautics, Navy Department, the former for guidance in details relating to practical requirements of the system, the latter for being the first, to our knowledge, to suggest the idea of an automatic weather station for remote location. Acknowledgment is also made to Julien P. Friez & Son, Inc., for cooperation in the selection of weather instruments for this development and in furnishing these instruments.

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EFFECT OF OXYGEN AND MOISTURE ON THE STABILITY OF LEATHER AT ELEVATED TEMPERATURES

By Joseph R. Kanagy

ABSTRACT

A commercial leather and leathers tanned with sumac, chestnut, quebracho, cutch, and a blend of chestnut, sulfited quebracho, and cutch, were heated to temperatures from 67° to 130° C in helium, in air, and in oxygen, at atmospheric pressure in the absence of water vapor. The effect of varying the concentration of water vapor was studied with chestnut and quebracho leathers at 100° C. The loss of tensile strength and the increase in the nitrogenous materials extractable in 0.1 *N* sodium carbonate solution were used as measures of the deterioration, the former being regarded as the more reliable. Raising the temperature increased the deterioration under all circumstances. In air, deterioration was more rapid than in the inert gas (helium), and in oxygen it was more rapid than in air. The logarithm of the fractional deterioration of the dry leather increased linearly with the duration of heating in air. The effect of humidity was small for samples having a pH greater than 3 but increased rapidly at lower values of the pH. The most important features of an accelerated-aging test for leather are an elevated temperature and an oxidizing atmosphere containing moisture.

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I. INTRODUCTION

For the past few years, a series of investigations has been in progress at the National Bureau of Standards relating to the various factors which affect the aging qualities of leather. In the course of that work,[1,2] to develop a suitable accelerated aging test, leather aged under normal conditions was compared with similar leather aged at elevated temperatures in an atmosphere of oxygen under pressure in a bomb. When vegetable-tanned leather was exposed in an oxygen bomb under the accelerated-aging conditions, the deterioration paralleled that obtained during normal aging with respect to change of rate of deterioration with pH [3] and the influence of the tanning materials on the stability of the leather. The effects of varying the atmosphere, time, temperature, and moisture contents were also studied.

[1] J. Research NBS **18**, 713 (1937) RP1004; J. Am. Leather Chem. Assn. **32**, 314 (1937).
[2] J. Research NBS **21**, 241 (1938) RP1128; J. Am. Leather Chem. Assn. **33**, 565 (1938).
[3] J. Am. Leather Chem. Assn. **30**, 370 (1935).

Although aging of leather in the bomb showed considerable promise as an accelerated-aging test, several disadvantages of using this method were evident. The operation of a test under pressure requires special equipment, and the deterioration in the bomb was not caused entirely by the oxygen and elevated temperature but also by the moisture which was formed from the oxidation of the tanning materials. This factor could not be conveniently controlled and made it difficult to compare the aging quality of a vegetable-tanned leather with a leather, such as chrome, which contains no oxidizable tanning materials.

In the present investigation a different apparatus was designed and a method of aging used which is believed to be an improvement over aging in the bomb. Further results were obtained on the effects of the various factors which influence the aging qualities of leather, and the essential features of an accelerated-aging test are more definitely established.

II. APPARATUS AND MATERIALS

The apparatus used in making the tests is shown in figure 1. The exposure chamber consists of a brass cylinder 7 in. in diameter and

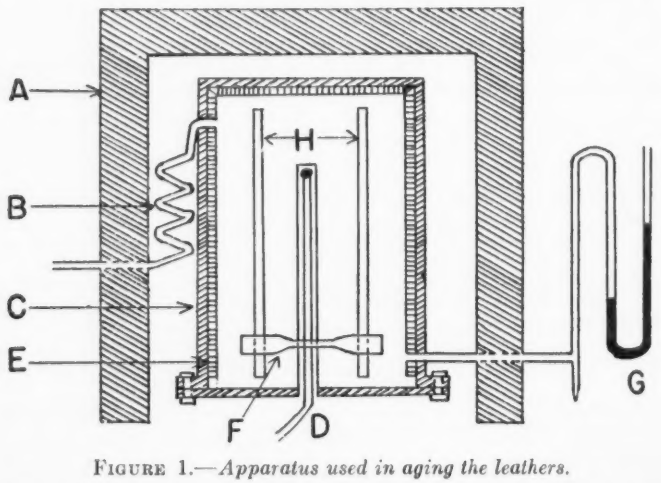

FIGURE 1.—*Apparatus used in aging the leathers.*

A, oven; *B*, pre-heating coil; *C*, exposure chamber; *D*, thermocouple; *E*, asbestos lining; *F*, leather sample; *G*, flowmeter; *H*, rack.

14 in. long. A removable cover is fitted on one end and fastened with bolts to a flange connected to the cylinder. The inside of the cylinder is lined with asbestos. The temperature inside the chamber is determined by means of a thermocouple placed inside a copper tube which protrudes into the cylinder. Inlet and outlet openings at opposite ends and opposite sides permit the flow of a continuous stream of gas. The rate of flow is determined by means of a flowmeter. A slightly increased pressure is required to operate the flowmeter, and therefore the pressure inside the chamber is about 10 mm above atmospheric pressure. The gases are preheated before entering the chamber by passing through the coil in the inlet tubing.

The apparatus used in regulating the amount of moisture in the gas stream is shown in figure 2. It consists of saturators filled with distilled water and placed in a water bath, the temperature of which is controlled within $\pm 0.1°$ C by means of a mercury regulator.

The leathers used in this work were sumac, chestnut, quebracho, cutch, a blend of chestnut, sulfited quebracho, and cutch, and a commercial leather. All except the commercial leather were tanned in the experimental tannery at the National Bureau of Standards.

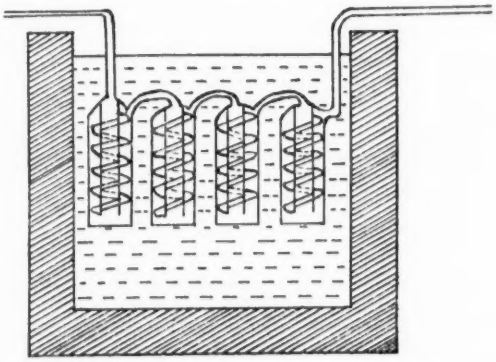

FIGURE 2.—*Water bath and saturators used to regulate the humidity of the gas streams.*

III. PROCEDURE

Twelve tensile-strength specimens were cut from blocks of the leathers 7 by 14 ins. These were numbered 1 to 12 consecutively. The even-numbered specimens were aged while the odd-numbered ones were used as controls. The samples were laid on racks in the exposure chamber to age. In order to assure uniform moisture contents for the leathers, it was found necessary to introduce an initial drying period. This was done by allowing the samples to condition at the same temperature at which they were to be aged, for 6 hours in a stream of dry gas before placing the cover on the chamber. At the end of this period of time, the cover was placed on the chamber and the flow of gas regulated to 8 to 12 ml/min.

Percentage loss in tensile strength and the percentage of the total nitrogenous materials extractable were used to evaluate the deterioration of the leathers. In recording the results of the tensile-strength measurements, the average value for the six specimens was used in each experiment. All samples were conditioned for 48 hours and tested at 65-percent relative humidity. After the breaking-strength measurements were made, the broken specimens were ground and a determination of the extractable nitrogenous material was made by extracting a sample with a solution of 0.1 *N* sodium carbonate.[4]

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The changes caused by varying the following conditions were studied: the temperature, the gas in contact with the leathers, the acidity of the leathers, the humidity, and the duration of exposure.

[4] J. Research NBS **17**, 247 (1936) RP909; J. Am. Leather Chem. Assn. **32**, 12 (1937).

In all experiments, except those in the study of the effect of duration of exposure, the time of aging was 7 days.

1. EFFECT OF TEMPERATURE AND ATMOSPHERE

The leathers were subjected to dry gases at 67° to 130° C to determine the effects of heat and oxidation. These temperatures were the exact readings of the thermocouples inside the exposure chamber and were slightly higher than the thermometer readings of the oven.

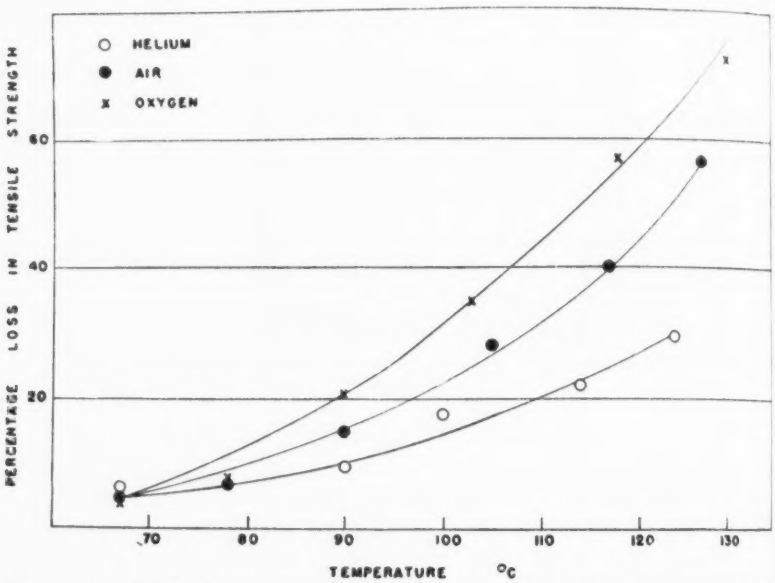

FIGURE 3.—*Effect of temperature and aging atmosphere on leather as shown by the percentage loss in tensile strength.*

(Average results for all the leathers exposed.)

The study was made by exposing the leathers for 7 days in helium, in air, and in oxygen. The pH values of all the leather samples were above 3, and therefore the effect of acidity was negligible. The gases were dried before passing over the leathers, and further precautions to assure dry conditions were taken by placing a dish containing calcium oxide in the exposure chamber. The results are given in table 1 and in figures 3, 4, and 5. For the purpose of comparing the effects of the three different atmospheres, the results for all the leathers exposed under each condition were averaged and are plotted in figures 3 and 4.

Beginning at about 90° C, a rapid acceleration with increase in temperature may be observed. The deterioration in the presence of the inert helium gas may be considered as that caused by heat alone.

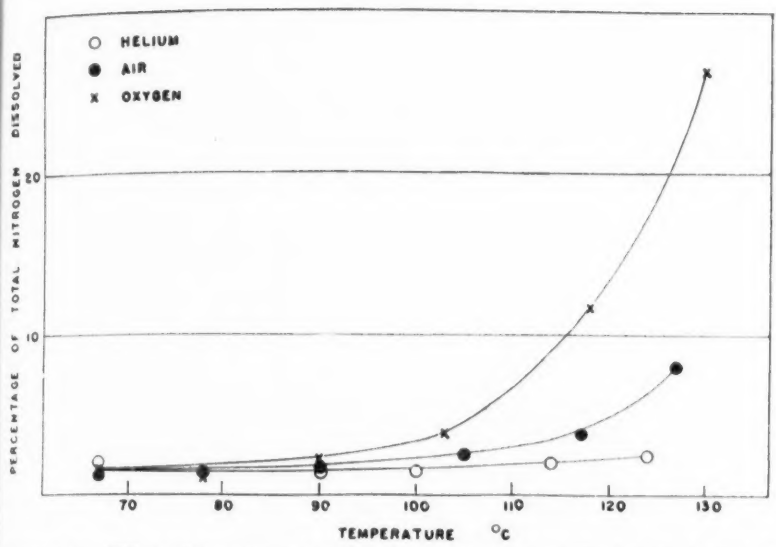

FIGURE 4.—*Effect of temperature and aging atmosphere on leather as shown by the percentage of the total nitrogen extracted.*

(Average results for all the leathers exposed.)

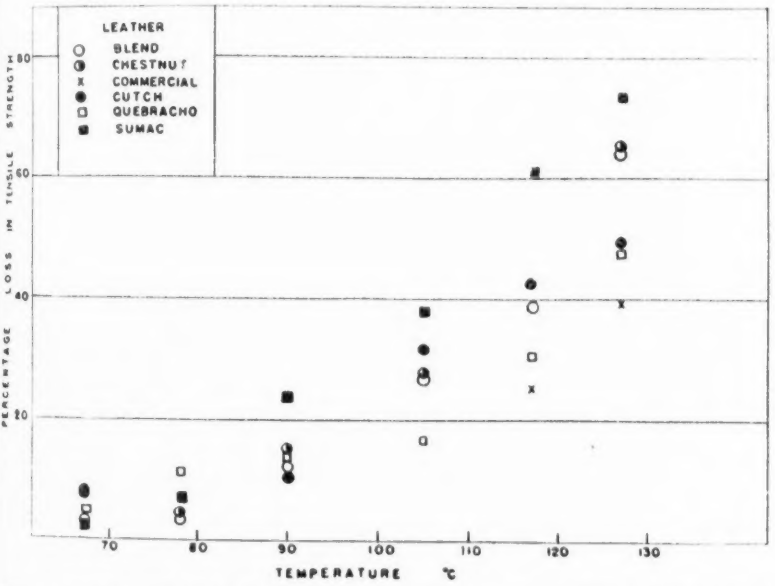

FIGURE 5.—*Increase in deterioration with increase in temperature of leathers aged in air.*

TABLE 1.—*Effect of temperature and atmosphere on leather*

Temperature, °C			67°		90°		100°		114°		124°			
Leather	Aging atmosphere	pH	Loss in tensile strength	Total nitrogen extracted	Loss in tensile strength	Total nitrogen extracted	Loss in tensile strength	Total nitrogen extracted	Loss in tensile strength	Total nitrogen extracted	Loss in tensile strength	Total nitrogen extracted	Loss in tensile strength	Total nitrogen extracted
			%	%	%	%	%	%	%	%	%	%	%	%
Blend	Helium	4.00	6.1	1.1	13.3	1.4	20.4	1.3	26.6	1.7	27.1	2.0		
Chestnut	do	4.28	7.8	1.8	10.8	2.0	21.6	1.8	21.1	2.6	35.1	3.4		
Commercial	do	3.10					10.5	2.5	17.3	2.7	17.7	2.7		
Cutch	do	4.93	11.7	1.4	9.1	1.2	18.9	1.1	23.6	2.0	34.9	2.3		
Quebracho	do	5.17	5.8	1.4	3.8	0.95	12.9	1.1	12.5	1.4	23.5	3.0		
Sumac	do	3.78	0.5	2.0	11.9	1.1	22.7	1.3	31.4	1.6	39.6	1.7		
Temperature, °C			67°		78°		90°		105°		117°		127°	
Blend	Air	4.00	2.7	1.1	4.8	1.4	12.6	1.1	27.2	2.0	38.9	3.7	64.6	8.1
Chestnut	do	4.28	7.9	1.5	5.3	1.7	15.5	2.5	28.0	5.4	43.0	6.3	64.9	10.5
Commercial	do	3.10									25.1	4.5	39.5	8.9
Cutch	do	4.93	7.6	1.2			11.1	1.8	32.0	2.5	43.0	4.0	49.7	8.9
Quebracho	do	5.17	4.6	0.8	11.2	1.1	13.1	1.3	16.8	1.6	30.9	2.3	47.9	5.4
Sumac	do	3.78	1.9	1.3	6.9	1.3	23.7	1.5	37.8	1.6	61.0	2.3	73.3	6.3
Temperature, °C			67°		78°		90°		103°		118°		130°	
Blend	Oxygen	4.00	4.5	0.9	10.4	0.9	17.7	2.0	26.5	3.9	50.4	11.8	71.1	28.5
Chestnut	do	4.28	6.9	1.6	5.8	1.7	23.2	4.0	33.2	6.4	60.8	17.1	73.4	30.8
Commercial	do	3.10	−1.0	2.2	0.2	2.2	12.6	2.7	25.1	4.9			64.3	33.4
Cutch	do	4.93	5.0	1.4	11.3	1.4	30.2	1.6	51.1	2.8	58.2	10.5	67.8	23.0
Quebracho	do	5.17	−2.2	0.95	5.1	0.8	10.8	1.3	27.2	2.9	48.1	10.5	71.9	22.0
Sumac	do	3.78	9.4	1.5	11.9	1.0	30.8	1.7	47.7	2.2	68.0	8.2	85.3	20.5

The accelerating effect of oxygen on the deterioration of the leather is clearly shown in figures 3 and 4, where considerably more deterioration is evident in oxygen or air than in helium. The increased deterioration where oxygen is present is, however, not proportional to the increase in its partial pressure. Similar results were observed in the study of the deterioration occurring in the oxygen bomb. High concentrations of oxygen therefore appear to accelerate deterioration little more than moderate concentrations and the rate of deterioration appears to be more effectively accelerated by raising the temperature. The loss in tensile strength with increase in temperature for all the leathers aged in an atmosphere of air is shown in figure 5. This figure shows the separation of the points representing the percentage losses in the tensile strength of the leathers, which indicates another advantage of using an elevated temperature for an aging test.

The degree of tannage appears to be an important factor in determining the stability of the leathers under these conditions. Sumac leather, which was least stable, had the lowest degree of tannage, while the commercial and quebracho leathers, which had the highest degrees of tannage, were most stable.

The results for percentage loss in tensile strength and percentage of the total nitrogenous materials dissolved, as shown in table 1, do not give the same comparative values for the stability of the leathers. From the values of the degrees of tannage, from physical examination, and previous results, loss in tensile strength appears to be a more reliable criterion of deterioration than the percentage of the nitrogenous materials extractable.

2. EFFECT OF DURATION OF EXPOSURE

The effect of duration of exposure was studied in an atmosphere of dry air at 110° C with chestnut- and quebracho-tanned leathers having pH values above 3.5. The results are shown in figure 6, where the logarithm of the percentage of tensile strength retained by the leathers after aging is plotted against the time in days. This relation is linear for the dry leathers during the regular aging period. The lines, however, do not coincide at log 2 because of the greater rate of deterioration which occurs during the initial drying period. If it is assumed that the percentage loss in tensile strength of the leather is proportional to the percentage of the total number of structural linkages broken then the linear relationship shown in figure 6 indicates that the reaction which causes the loss in tensile strength is one of the first order. The straight-line relationship also offers the possibility of making long-term predictions of the durability of leathers.

3. EFFECT OF VARYING THE HUMIDITY

The effect of varying the humidity was studied on samples of chestnut- and quebracho-tanned leathers and on similar leathers containing varying amounts of sulfuric acid. The pH values of the samples ranged from about 5.4 to 1.9. The leathers were aged at 100° C for 1 week, and the humidity was controlled by saturating the gas stream with water at a definite temperature. The gas was passed through the saturators, which were held at a constant temperatu e in

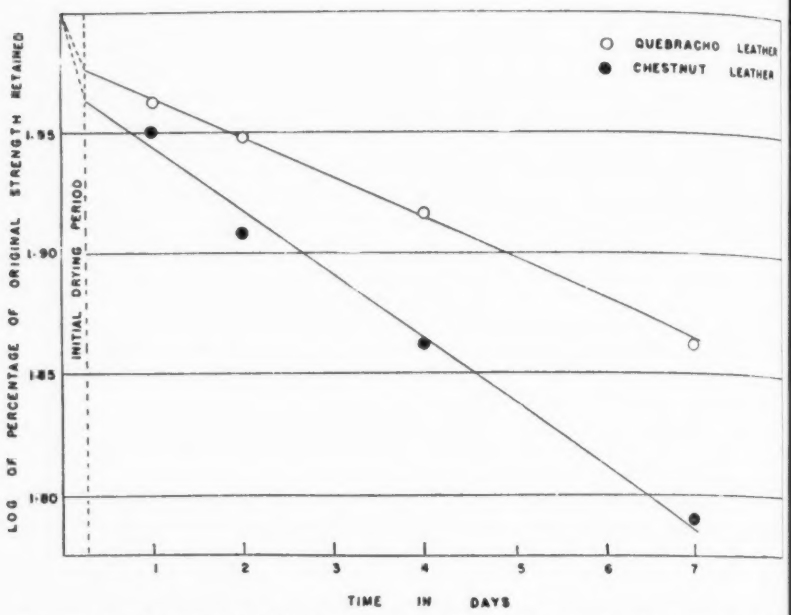

FIGURE 6.—*Increase in deterioration of chestnut- and quebracho-tanned leathers with increase in time of aging in air at 110° C.*

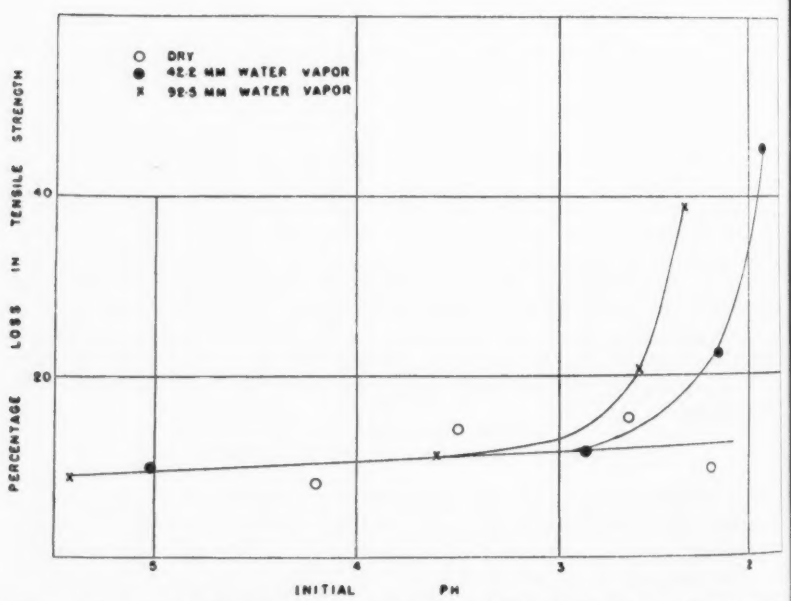

FIGURE 7.—*Effect of increasing the moisture contents of the aging atmosphere for quebracho leather aged in helium at 100° C.*

the water bath. By raising the temperature of the water bath, the amount of moisture in the gas stream could be increased. In the presence of helium, the pressure of water in the gas stream was varied from approximately 0 to 92.5 mm. The variation over this range was considered sufficient to show the accelerating effect of water vapor. These partial pressures of water in terms of relative humidity at 100° C are small but were sufficient to show a considerable effect on the rates of deterioration. The results are shown in table 2 and in figures 7 and 8. Above pH 3 little or no effect is shown on the

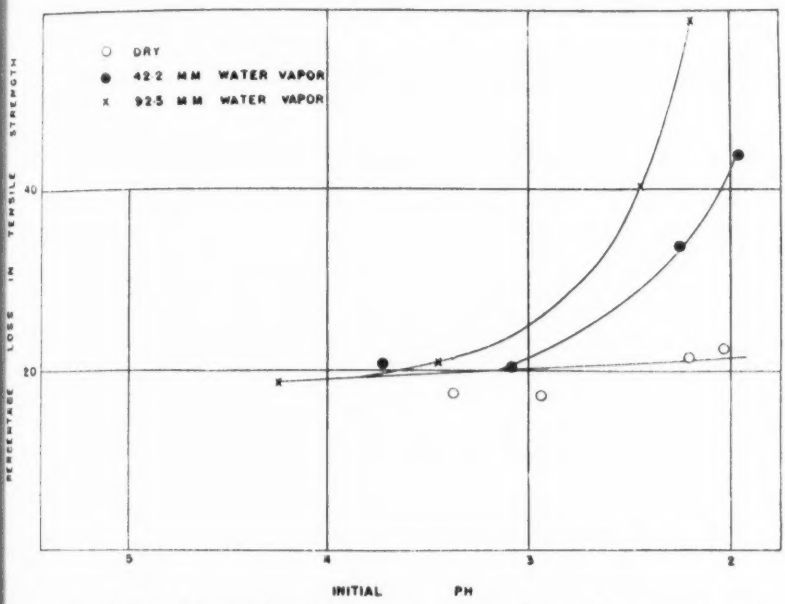

FIGURE 8.—*Effect of increasing the moisture contents of the aging atmosphere for chestnut leather aged in helium at 100° C.*

rates of deterioration when the amounts of moisture are increased throughout the range of these experiments. However, below pH 3 a considerable effect may be observed with definite indications that, in the absence of moisture, acid has little effect on the rate of deterioration.

Experiments were also made with air and with oxygen in which the water vapor pressures in the gas streams were 31.8 and 55.3 mm. These results are also shown in table 2. A comparison of the results obtained for helium, air, and oxygen on quebracho-tanned leather in which the vapor pressure of water was 31.8 mm is shown in figure 9.

TABLE 2.—*Effect of humidity on leather at 100° C*

Vapor pressure of water in aging atmosphere, mm—		0			31.8			42.2			55.3			92.5		
Leather	Aging atmosphere	pH	Loss in tensile strength	Total nitrogen extracted	pH	Loss in tensile strength	Total nitrogen extracted	pH	Loss in tensile strength	Total nitrogen extracted	pH	Loss in tensile strength	Total nitrogen extracted	pH	Loss in tensile strength	Total nitrogen extracted
			%	%		%	%		%	%		%	%		%	%
Quebracho	Helium	4.20	8.2	0.9	5.07	8.7	1.3	5.03	10.0	1.0	5.05	10.3	1.3	5.42	8.8	1.3
Do	do	3.50	14.1	1.3	3.00	10.3	1.3	2.87	11.3	1.5	2.97	18.9	1.3	3.60	11.2	1.1
Do	do	2.65	15.5	1.1	1.90	37.3	22.8	2.18	22.6	6.1	2.12	28.0	6.8	2.60	20.7	3.4
Do	do	2.23	9.9	1.6	1.88	37.4	26.1	1.88	45.3	27.8	1.84	53.4	40.8	2.36	39.0	11.1
Chestnut	do	3.37	17.3	1.8	3.74	10.1	2.7	3.73	20.8	3.3	3.70	15.6	3.8	4.25	18.5	3.7
Do	do	2.93	17.1	2.1	3.15	24.1	4.1	3.09	20.1	5.1	3.13	22.1	3.9	3.49	20.8	3.8
Do	do	2.20	21.5	2.4	2.39	23.3	7.5	2.25	33.7	17.3	2.25	26.6	14.8	2.45	40.4	25.2
Do	do	2.03	22.4	4.4	1.90	45.7	40.5	1.97	43.9	40.2	1.93	56.0	60.3	2.19	59.1	68.2
Quebracho	Air				5.13	12.1	1.4				5.00	9.3	1.5			
Do	do				3.50	23.4	1.4				3.00	13.5	1.5			
Do	do				2.22	26.6	4.6				2.03	37.5	16.9			
Do	do				2.05	32.5	13.0				1.88	55.3	39.5			
Chestnut	do				4.03	23.8	3.4				3.70	17.6	4.0			
Do	do				3.32	19.8	4.4				3.12	37.0	5.8			
Do	do				2.33	42.6	17.2				2.45	31.8	14.6			
Do	do				2.12	44.7	38.7				1.98	55.4	62.5			
Quebracho	Oxygen				5.17	16.3	1.7				5.17	19.2	2.4			
Do	do				3.37	21.7	2.2				3.47	18.6	2.3			
Do	do				2.23	35.2	9.5				2.57	24.7	5.7			
Do	do				1.93	52.3	37.3				2.22	32.9	13.4			
Chestnut	do				3.93	28.6	6.6				4.22	32.3	11.5			
Do	do				3.33	34.4	9.3				3.36	36.6	15.0			
Do	do				2.35	51.9	42.7				2.48	54.8	56.1			
Do	do				2.07	63.3	83.4				2.25	58.7	80.5			

The accelerating effect of the oxygen is shown here throughout the entire pH range. The differences between the percentage losses in tensile strength in the presence of oxygen and in the presence of helium under these conditions are approximately the same as those under the dry conditions. These results are similar to those obtained

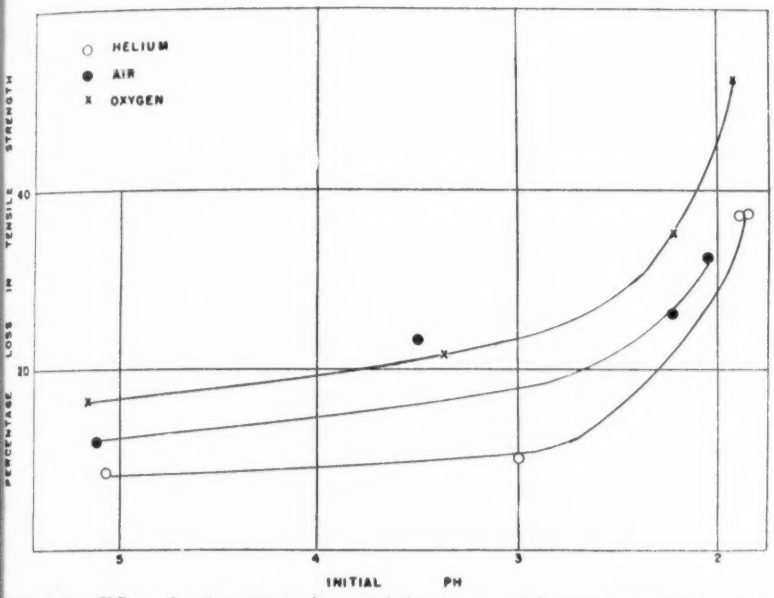

FIGURE 9.—*Effect of aging atmosphere of helium, air, and oxygen containing 31.8 mm of water vapor on leather at different* pH *values.*

under normal aging conditions with respect to change in rate of deterioration with decrease in pH.

V. CONCLUSIONS

Deterioration of leather as measured by percentage losses in tensile strength and percentage of the total nitrogenous materials extractable is accelerated by oxygen at elevated temperatures. Increased temperature alone, as shown in an inert helium atmosphere, also accelerates deterioration. The effect of humidity on the rate of deterioration is greatest below pH 3.

These results indicate that the most important features of an accelerated-aging test for leather are aging in an oxidizing atmosphere containing moisture, at an elevated temperature. This is the same principle as that involved in the oxygen-bomb test. The method of applying the test described in this paper is however believed to be an improvement over the oxygen-bomb test, since it permits a more convenient control of all the factors involved.

WASHINGTON, May 21, 1940.

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PHYSICAL, MINERALOGICAL, AND DURABILITY STUDIES ON THE BUILDING AND MONUMENTAL GRANITES OF THE UNITED STATES

By Daniel W. Kessler, Herbert Insley, and William H. Sligh

ABSTRACT

Tests on 116 samples of granite (including the "black granites") from the principal producing districts of the United States gave range values for various physical properties, as follows: Compressive strength, 7,700 to 53,800 lb/in.^{2}; absorption (48-hr immersion), 0.02 to 0.58 percent; porosity, 0.40 to 3.84 percent; weights per cubic foot, 157 to 187 lb. The "black granites" (gabbros and basalts) were indicated to be stronger, denser, and less absorptive than the normal granites. Tests on a few selected samples gave a range of values for shearing strength of 3,900 to 4,600 lb/in.^{2}; modulus of rupture, 1,430 to 5,190 lb/in.^{2}; abrasive hardness, (H_a) 37 to 88; toughness, 8 to 27.

In compression, granite is indicated to be stronger than other types of masonry materials except for some quartzites, whereas slate and some of the serpentines are stronger than granite in flexure. The porosity of granite is of the same order as that of the marbles, slates, and quartzites, but much lower than that of most other masonry materials. Although granite ranks as a heavy masonry material, the unit weight of a normal granite is somewhat less than that of marble, slate, and serpentine.

Inspections of granite structures have shown several cases in which the granite scaled to a depth of a few hundredths of an inch over a portion of the surface. Chemical and microscopic studies showed that calcium sulfate was present in the decayed granite but not in newly quarried granite. Experiments were made to determine if calcium sulfate, leached into the granite from external sources or formed in the granite by the action of acids of sulfur on calcite ingredients, would produce scaling. Leaching with a 10-percent solution of sulfuric acid caused some granites to disintegrate within 30 days, but leaching with a saturated solution of calcium sulfate produced no decay in 3 years. It was concluded that the conversion of calcite ingredients to calcium sulfate by acids of sulfur is the probable cause of scaling.

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I. INTRODUCTION

This study of granites is a continuation of the program [1] for determining properties of the building and monumental stones which may be of interest in the selection of materials for specific uses.

One hundred and sixteen samples were collected, and petrographic analyses were made on those granites which had not been previously studied for mineral composition. Tests for compressive strength, porosity, absorption, and density were made on most of the samples to afford comparisons between the various granites. A few selected samples were tested for shearing strength, modulus of rupture, abrasive hardness, toughness, and elasticity, in order to show how these properties for granite compare with those of other masonry materials. Certain durability studies were made to determine what agents are mainly concerned in the weathering of granite.

Since there were only a few samples from 12 States, the results for these are grouped under "Miscellaneous Granites" and those for the 10 main producing States are tabulated separately in geographic order. For convenience in comparing different granites, the results for compressive strength, absorption, density, and porosity have been assembled in one table. Data on other masonry materials, given for comparative purposes in the discussion of different physical properties, were obtained from various sources.

In a study of this kind an important consideration is how well the samples represent the deposits from which they were taken. Each producer was relied upon to select and furnish representative samples of his granite. It should not be assumed, however, that any deposit is uniform throughout. Probably granite from all deposits varies in quality with depth and distance from open joints. The samples used in this study are assumed to be near the average of the products from the quarries represented.

II. SAMPLES AND TEST SPECIMENS

The samples were 6 by 6 by 24 in. and were selected by the producers to represent the average product of their quarries. In a few instances two or more samples were submitted from the same quarry to show a range in texture or color. No special finish was required, and most of the samples were split from larger blocks without any further surfacing.

[1] D. W. Kessler, *Physical and Chemical Tests on the Commercial Marbles of the United States*, Tech. Pap. BS **12** (1919) T123.
D. W. Kessler, *Permeability of Stone*, Tech. Pap. BS **20**, 155 (1925) T305.
D. W. Kessler and W. H. Sligh, *Physical Properties of the Principal Commercial Limestones Used for Building Construction in the United States*, Tech. Pap. BS **21**, 497 (1927) T349.
D. W. Kessler and W. H. Sligh, *Physical Properties and Weathering Characteristics of Slate*, BS J. Research **9**, 377 (1932) RP477.
D. W. Kessler, *Wear Resistance of Natural Stone Flooring*, BS J. Research **11**, 635 (1933) RP612.
D. W. Kessler, *Action of Hypo Solution on Stone Tanks*, J. Research NBS **16**, 161 (1936) RP863.

Others were sawed and a few were polished on one or more faces. The rift faces on some of the samples were not marked by the producers, and for these the rift was judged by inspection or by simple tests.

Five producers supplied prepared specimens for flexure tests. Other specimens for the different tests were prepared at the National Bureau of Standards. For most tests these consisted of cores approximately 2.1 in. in diameter and 2.5 in. high. These were cut with core drills using No. 60 silicon carbide and water. The cores were cut to a depth of 3 in., broken out, and finished on the ends by grinding. Seven cores from each sample with the axes parallel to the rift and 7 with the axes perpendicular to the rift were prepared, as indicated in figure 1.

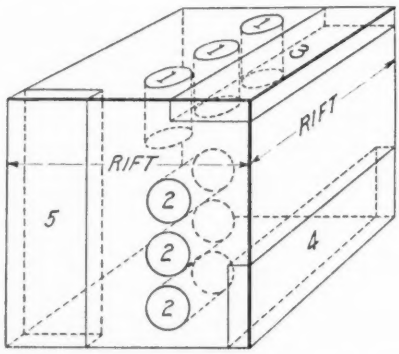

FIGURE 1.—*Diagram showing manner of preparing specimens with respect to rift direction.*

Cores marked "1" are cut perpendicular to rift and cores marked "2" are parallel to rift. Flexural test specimens 3 and 4 are for test perpendicular to rift and 5 parallel.

The ends of specimens used in the compression tests were finished with special care.

The specimens for flexure tests were 12 by 4 by 1 in., and, in these tests, they were broken in two equal parts. These parts served for shearing tests without further modification. The 2- by 2- by 1-in. specimens used for abrasive hardness tests were, in most cases, prepared from the ends of flexure test specimens, but in some cases they were sawed out of larger blocks with silicon-carbide wheels. The specimens for toughness tests were 2.5 cm (approximately 1 in.) in diameter by 2.5 cm high.

III. DESCRIPTION OF SAMPLES

The granites described in table 1 are assigned serial numbers, which are used to identify them in succeeding tables. The mineral constituents and mineral alterations are based on microscopic studies made at this Bureau or at other laboratories, as shown in column 12.

TABLE 1.—*Description of*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of
b=porosity, *c*=absorption, *d*=water transmission rate, *e*=deterioration in the soaking and drying test.
"—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Vinalhaven, Maine.	Biotite granite	Gray	Fine to medium.	Orthoclase, quartz, oligoclase, and biotite.	
2	do	Olivine norite	Nearly black.	Fine grained.	Soda-lime feldspars (labradorite to bytownite), hypersthene, olivine, and magnetite.	
3	do	Biotite granite	Pink	Coarse grained.	Orthoclase and microcline, quartz, oligoclase, biotite.	Magnetite, zircon, and apatite.
4	do	do	do	do	do	do
5	do	Biotite hornblende granite.	Gray	Fine grained.	Orthoclase and microcline, smoky quartz, oligoclase, biotite, hornblende.	Magnetite, titanite, and apatite.
6	do	Biotite granite	Pinkish gray.	Coarse grained.	Orthoclase and microcline, quartz, oligoclase, and biotite.	Magnetite, zircon, and apatite.
7	Mt. Desert, Maine.	do	Gray with tinge of pink.	Medium grained.	Orthoclase, plagioclase, smoky quartz, oligoclase, and biotite.	Apatite and calcite.
8	do	do	Gray	Fine grained.	Orthoclase, smoky quartz, oligoclase, and biotite.	Magnetite.
9	Jonesboro, Maine.	do	Pinkish gray.	Medium to coarse.	Orthoclase, smoky quartz, oligoclase, and biotite.	Magnetite, muscovite, zoisite, and chlorite.
10	Long Cove, Maine.	Biotite-muscovite granite.	Bluish gray.	Fine to medium.	Microcline and orthoclase, quartz, oligoclase, biotite, and muscovite.	Garnet, zircon, apatite, and chlorite.
11	North Jay, Maine.	Biotite-muscovite granite.	Light gray.	Fine grained	do	Garnet, magnetite, and apatite.
12	Stonington, Maine.	Biotite granite	Gray with lavender tint.	Coarse grained.	Orthoclase and microcline, smoky quartz, oligoclase, and biotite.	Muscovite, magnetite, titanite, zircon, and pyrite.
13	Frankfort, Maine.	Biotite granite	Gray	Medium	do	Magnetite and titanite.
14	Lincolnville, Maine.	Muscovite-biotite granite.	Bluish gray.	Fine grained.	Orthoclase and microcline, smoky quartz, oligoclase, muscovite, and biotite.	Apatite
15	St. George, Maine.	Biotite-muscovite granite.	do	Fine to medium.	Microcline and orthoclase, quartz, oligoclase, biotite, and muscovite.	Garnet, zircon, apatite, and chlorite.

See footnotes at end of table.

granites and their uses

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength. The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (or explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Oligoclase partly altered to kaolin and white mica.	0.56	*a*+, *b*, *c*, *d*−, *e*−	Paving blocks	A	Some porphyritic feldspars.
	3.20	*a*+, *b*−, *c*−, *d*−	Monumental	A	Material takes a high polish.
Oligoclase partly altered to kaolin and white mica.	0.31	*a*−, *b*+, *c*, *d*+, *e*−	Constructional and architectural [1]	A	Considerable reduction in strength for wet specimens.
	.49	*a*, *b*−, *c*−, *d*−	do.[2]	A	(1860.)
Orthoclase slightly altered to kaolin and white mica.	.38	*a*+, *b*−, *c*−, *d*−	Paving blocks	A	Porphyritic feldspars up to ¼ in. (1887.)
			do	A	(1860.)
Some secondary calcite in the oligoclase.	.25	*a*+, *b*+, *e*−, *d*−, *e*−	Architectural and constructional [3]	A	Sample showed large reduction in strength for wet condition. (1880).
do	.25	*a*+, *b*−, *c*, *d*−, *e*−	Paving blocks	A	Sample has porphyritic feldspars over ¼ in. in length.
Oligoclase considerably altered to kaolin and white mica; biotite to chlorite.	.79	*a*, *b*+, *c*, *d*−, *e*+	Architectural and constructional [4]	A	Smoky quartz areas are prominent.
	1.45	*a*, *b*−, *c*, *d*−, *e*−	do.[5]	A	The sample showed a considerable reduction in strength for the wet condition. (1873.)
	0.36	*a*, *b*, *c*+, *d*−, *e*+	Architectural and monumental [6]	A	Strength values were rather variable. (1872.)
Oligoclase much altered to kaolin and white mica.	.49	*a*−, *b*, *c*, *d*−	Architectural [7]	A	(1872.)
Secondary chlorite kaolin and white mica.	.32	*a*+, *b*, *c*+, *d*+, *e*−	do.[8]	A	The large water transmission value indicates open pore structure. (1851.)
do		*a*−, *c*+	Monumental	A	(1875.)
Oligoclase partly altered to white mica.	.25	*a*−	Architectural and monumental. [9]	A	(1870.)

TABLE 1.—*Description of granites*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of b=porosity, c=absorption, d=water transmission rate, e=deterioration in the soaking and drying test. "—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
16	Highpine, Maine.	Biotite granite	Pink	Coarse	Microcline and orthoclase, smoky quartz, oligoclase, and biotite.	Magnetite, zircon, muscovite, and chlorite.
17	Concord, N. H.	Muscovite-biotite granite.	Bluish gray.	Fine to medium.	Microcline and orthoclase, quartz, oligoclase-albite, muscovite, and biotite.	Magnetite, apatite, zircon, and rutile.
18	Redstone, N. H.	Biotite granite	Pink	Coarse grained.	Orthoclase, smoky quartz, (oligoclase-albite), and biotite.	Magnetite, pyrite, apatite, fluorite, and zircon.
19	do	Biotite-hornblende granite.	Dark green.	Coarse	Orthoclase, smoky quartz, plagioclase (albite-oligoclase), biotite, and hornblende.	Magnetite, fluorite, allanite, and zircon.
20	Mason, N. H.	Quartz-biotite-monzonite.	Gray	Medium grained.	Potash feldspar, quartz, plagioclase, and biotite.	Muscovite, magnetite, and epidote.
21	Milford, N.H.	Quartz-monzonite.	Light gray.	Fine grained	Microcline and orthoclase, oligoclase, smoky quartz, and biotite.	Allanite, zircon, and apatite.
22	do	do	Dark gray	do	Smoky quartz, oligoclase, microcline, and biotite.	Magnetite, pyrite, apatite, allanite, and zircon.
23	do	do	Bluish gray.	Medium grained.	Smoky quartz, oligoclase, microcline, and biotite.	Magnetite, apatite, pyrite, allanite, and zircon.
24	Fitzwilliams, N. H.	Biotite-muscovite granite.	do	Fine grained	Orthoclase and microcline, quartz, oligoclase, biotite, and muscovite.	Rutile
25	Barre, Vt.	Biotite granite	do	do	Orthoclase, smoky quartz, (oligoclase-albite), biotite, and muscovite.	Titanite, magnetite, and pyrite.
26	do	do	do	Medium	Orthoclase and microcline, smoky quartz, oligoclase-albite, biotite, and muscovite.	Magnetite and zircon.
27	Derby, Vt.	Quartz monzonite.	do	Fine to medium.	Smoky quartz, oligoclase, microcline and orthoclase, biotite, and muscovite.	Apatite, titanite, allanite, and rutile.
28	Woodbury, Vt.	Biotite granite	Light gray.	Coarse	Microcline and orthoclase, quartz, oligoclase, biotite, and muscovite.	
29	do	do	Bluish gray.	Medium	do	Pyrite, apatite, zircon, and rutile.

See footnotes at end of table.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength, The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (for explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Oligoclase partly altered to white mica; biotite to chlorite.	0.63	*a*, *c*+, *d*+, *e*	Architectural [10]	A	Sample contains feldspars up to ½ in. in length.
Secondary kaolin, white mica, and calcite.	.86	*a*−, *b*+, *c*+, *d*+, *e*−.	do.[11]	A	Strength values were rather variable. (1863.)
Secondary kaolin, chlorite, titanite, and calcite.	.36	*a*−, *b*, *c*, *d*+, *e*−	do.[12]	A	Sample contained feldspars more than 1 in. in length. (1887.)
Secondary kaolin, white mica, calcite, epidote, chlorite, and limonite.	.58	*a*, *b*−, *c*−, *d*	Building and ornamental stone.[13]	A	Compressive strength values were variable. (1887.)
Slight alteration of orthoclase to sericite and biotite to chlorite.	.58	*a*, *b*, *c*+, *d*−, *e*+	Architectural and constructional.	B	The potash feldspar consists of both microcline and orthoclase, the latter sometimes containing inclusions of quartz.
Secondary chlorite, white mica, kaolin, carbonate, and hematite.	.50	*a*−, *b*, *c*+, *d*+, *e*	do.[14]	A	(1886.)
Secondary kaolin, white mica, chlorite, carbonate, and hematite.	.....	*a*, *c*	Monumental	A	
do	.....	*a*+, *c*+	do	A	(1876.)
Secondary chlorite and kaolin.	.36	*a*, *b*, *c*, *d*+	do.[15]	A	Nearly all of the minerals are less than 1/10 in. in size. (1904.)
Secondary calcite, kaolin, and white mica.	.80	*a*−, *b*−, *c*−, *d*−, *e*−.	do.[16]	A	Sample showed a high resistance in the crystallization test. (1887.)
Secondary calcite, kaolin, white mica, and chlorite.	.....	*a*, *c*	do.[17]	A	
Secondary kaolin, white mica, and calcite.	.31	*a*−, *b*, *c*, *d*+, *e*−	Architectural and monumental.[18]	A	(1880.)
Kaolin, carbonate, and white mica.	.14	*a*+, *b*+, *c*, *d*−, *e*−.	Architectural [19]	A	The sample contained very little mica. (1902.)
Kaolin and zoisite	.49	*a*−, *b*, *c*, *d*−	Architectural and monumental.[20]	A	(1896.)

TABLE 1.—*Description of granites*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of b=porosity, c=absorption, d=water transmission rate, e=deterioration in the soaking and drying test, "—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
30	Windsor, Vt.	Hornblende-biotite.	Dark	Fine to medium.	Orthoclase, smoky quartz, plagioclase (oligoclase-albite), hornblende, and biotite.	Magnetite, ilmenite, titanite, and allanite.
31	Beebe Plain, Vt.	Granodiorite	Light gray	Medium grained.	Plagioclase, quartz, microcline, and biotite.	Muscovite and pyroxene.
32	Dummerston, Vt.	Quartz monzonite.	Bluish gray.	Medium	Smoky quartz, oligoclase, microcline and orthoclase, muscovite, and biotite.	Apatite and rutile.
33	Quincy, Mass.	Riebeckite aegirite granite.	do	Coarse	Orthoclase, smoky quartz, riebeckite, aegirite, and albite.	Zircon, ilmenite, and pyrite.
34	do	do	do	do	do	do
35	do	do	do	do	Orthoclase, smoky quartz, albite, aegirite, and riebeckite.	Magnetite, zircon, and fluorite.
36	do	do	do	do	do	do
37	West Chelmsford, Mass.	Muscovite-biotite granite-gneiss.	Light gray	Medium	Microline and orthoclase, oligoclase, quartz, rutile, muscovite, and biotite.	Apatite.
38	do	do	do	do	do	do
39	Chester, Mass.	Biotite-muscovite granite-gneiss.	Bluish gray.	Medium to fine.	Microcline, quartz, plagioclase, biotite, and muscovite.	Pyroxene, titanite, apatite, and zircon.
40	Milford, Mass.	Biotite granite	Pale pink	Coarse	Microcline and orthoclase, quartz, plagioclase (albite to albite-oligoclase), and biotite.	Garnet, apatite, magnetite, pyrite, ilmenite, zircon, and allanite.
41	Rockport, Mass.	Hornblende granite.	Greenish gray.	do	Orthoclase and microcline, smoky quartz, hornblende, and plagioclase (albite-oligoclase).	Magnetite, molybdenite, fluorite, and zircon.
42	do	do	Olive green.	do	do	Magnetite, allanite, and zircon.
43	Ansonia, Conn.	Muscovite-biotite granite-gneiss.	Bluish gray.	Fine	Microcline and orthoclase, quartz, oligoclase, muscovite, and biotite.	Apatite and garnet.
44	do	do	do	do	do	do
45	East Lyme, Conn.	Quartz monzonite.	Golden pink.	do	Quartz, oligoclase, microcline and orthoclase, biotite, and muscovite.	Magnetite and apatite.
46	Waterford, Conn.	do	Buff	do	Oligoclase, microcline and orthoclase, quartz, biotite, and muscovite.	Magnetite, apatite, zircon, and allanite.

See footnotes at end of table.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength. The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (for explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Secondary kaolin	0.39	*a*+, *b*−, *c*−, *d*−	Architectural and monumental.[21]	A	Orthoclase is olive green; the other feldspars and hornblende are darker green.
Slight alteration in plagioclase.		*a*−, *c*	Monumental	B	
Secondary kaolin, white mica, epidote, zoisite, and calcite.		*a*, *c*+	Architectural and monumental.[22]	A	(1877.)
Leucoxene is secondary; riebeckite partly secondary.	.31	*a*, *b*+, *c*, *d*−, *e*	Monumental	A	Strength values are rather variable.
do	.41	*a*, *b*+, *c*, *d*−, *e*	do.[23]	A	(1846.)
Secondary kaolin, hornblende, and chlorite.	.31	*a*, *b*+, *c*, *d*+	do	A	
do	.32	*a*, *b*, *c*, *d*−	do	A	
Secondary kaolin, white mica, carbonate, epidote and chlorite.	.45	*a*, *b*+, *c*+, *d*	Architectural and constructional.[24]	A	The sample had a gneissic structure. (1881.)
do	.49	*a*, *b*+, *c*+, *d*+, *e*−.	do	A	Do.
Micas and pyroxene often corroded. Secandary calcite and kaolin.	1.17	*a*, *b*, *c*, *d*−	Monumental	B	
Secondary kaolin, white mica, epidote, zoisite, leucoxene, chlorite, and calcite.	0.31	*a*, *b*, *c*, *d*+, *e*−	Architectural and constructional.[25]	A	The sample has an appreciable gneissic structure.
Secondary kaolin and calcite.		*c*−	do.[26]	A	(1855.)
Secondary kaolin, chlorite, and limonite.		*c*−	do.[27]	A	(1868.)
Secondary kaolin and calcite.	.34	*a*−, *b*−, *c*, *d*−, *e*−.	Constructional	A	(1890.)
do	.41	*a*, *b*, *c*−, *d*−	do	A	(1890.)
Secondary hematite, kaolin, calcite, and white mica.	.58	*a*+, *b*+, *c*, *d*−, *e*+.	Monumental	A	(1900.)
Secondary white mica, calcite, chlorite, and limonite.	.56	*a*, *b*+, *c*+, *d*− *e*+.	Architectural and monumental.[28]	A	Strength values were quite variable. (1878.)

TABLE 1.—*Description of granites*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of b=porosity, c=absorption, d=water transmission rate, e=deterioration in the soaking and drying test. "—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Branford, Conn.	Biotite granite gneiss.	Reddish gray.	Medium to coarse.	Microcline and orthoclase, quartz, oligoclase, biotite, and muscovite.	Magnetite, pyrite, apatite, and zircon.
48	Stony Creek, Conn.	do	Reddish	Coarse	do	do
49	Duke, N. C.	Quartz-ilmenite schist	Dark gray	Fine grained	Quartz, ilmenite, pyroxene, amphibole, and hematite.	
50	do	do	do	do	do	do
51	Wake Forest, N. C.	Biotite granite	Gray	Fine to medium.	Orthoclase, microcline, acidic plagioclase, and biotite.	Muscovite, apatite, and zircon.
52	do	do	Bluish gray.	do	do	do
53	Mt. Airy, N. C.	do	Light gray.	Medium grained.	Oligoclase, orthoclase and microcline, quartz, and and biotite.	Apatite and zircon.
54	do	do	do	do	do	do
55	Salisbury, N. C.	do	Pink	do	Orthoclase with a small amount of microcline, plagioclase, quartz, and biotite.	Magnetite and garnet.
56	do	do	do	do	do	do
57	do	do	do	do	do	do
5	do	do	do	do	do	do
	Rion, S. C.	do	Light gray.	do	Orthoclase and microcline, plagioclase (oligoclase), quartz, and biotite.	Apatite, zircon, and iron oxide.
60	do	do	do	do	do	do
61	Newberry, S. C.	do	Bluish gray.	Fine grained.	Orthoclase with microcline, oligoclase, and biotite.	Apatite, zircon, and magnetite.
62	do	do	do	do	do	do
63	Greenwood, S. C.	do	Light gray.	Medium grained.	Microcline and orthoclase, oligoclase, quartz, and biotite.	Titanite, zircon, apatite, and iron oxide.
64	Winnsboro, S. C.	do	do	do	Orthoclase and microcline, plagioclase (oligoclase), quartz, and biotite.	Apatite, zircon, and iron oxide.

See footnotes at end of table.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a* = compressive strength. The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (for explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Secondary kaolin, white mica, calcite, epidote, zoisite, chlorite, and limonite.	0.59	*a, b, c, d*−	Architectural, constructional and monumental.[29]	A	(1887).
	.79	*a*−, *b*−, *c*−, *d*−	Architectural [30]	A	The sample showed a considerable reduction in strength for the wet specimens. (1876).
Minerals slightly altered by weathering.	.74	*a*+, *b*+, *c*−, *d*−	Architectural and constructional.	B	Splits rather easily in the direction of schistosity.
		a+, *b*−, *c*−, *d*−	do	B	Do.
Secondary chlorite, kaolin, and iron oxide.	.63	*a*, *b*−, *c*−, *d*+, *e*−.	do	C	Granite from this area is sometimes pinkish gray.
do	.43	*a*−, *b*, *c*−, *d*+	do	C	Do.
Secondary chlorite, epidote, mica, and iron oxide.	.22	*a*−, *b*−, *c*, *d*+, *e*−.	do [31]	C	Mica content is variable. (1889).
		a−, *b*+, *c*+, *d*, *e*−.	Architectural and constructional.	C	
Biotite is frequently altered.	.41	*a*+, *b*−, *c*−, *d*−	Architectural, constructional, and monumental.[32]	C	Some of the granite from this area is gray. (1875).
	.41	do	do [33]	C	Some of the granite from this area is gray. (1903).
	.67	*a*+, *b*−, *c*−, *d*−	do [34]	C	Smoky quartz grains up to 0.3 in.
	.67	*a*+, *b*, *c*−, *d*−	do	C	Smoky quartz is a prominent constituent.
Secondary chlorite, epidote, kaolin, and muscovite.	.79	*a*, *b*, *c*, *d*, *e*−	Architectural and constructional.[35]	C	There are intergrowths of the feldspars and of the feldspars with quartz. (1883).
		a, *c*+	do	C	
Secondary chlorite, epidote, and kaolin.	.49	*a*+, *b*+, *c*, *d*, *e*−	Architectural and monumental.	C	The orthoclase is intergrown with plagioclase.
do	.50	*a*+, *b*, *c*, *d*−	do	C	
Secondary chlorite, epidote, and muscovite.		*a*+, *c*+	Constructional	C	Much accessory titanite occurs in this granite.
Chlorite, epidote, and muscovite.	1.17	*a*, *b*, *c*, *d*+	Architectural and constructional.	C	

TABLE 1.—*Description of granite*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of
b=porosity, *c*=absorption, *d*=water transmission rate, *e*=deterioration in the soaking and drying test.
"—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
65	Elberton, Ga.	Biotite granite	Bluish gray.	Medium to fine grained.	Orthoclase and microcline, oligoclase, quartz, and biotite.	Apatite, zircon, and magnetite.
66	do	do	Pinkish gray.	do	do	do
67	Stone Mountain, Ga.	Muscovite granite.	Light gray.	Medium grained.	Orthoclase and microcline, plagioclase, quartz, muscovite, and biotite.	Black tourmaline, red garnet, apatite, and zircon.
68	Williamson, Ga.	Quartz monzonite.	Gray	do	Microcline, plagioclase, quartz, biotite, and muscovite.	Magnetite, apatite, and hypersthene.
69	Lithonia, Ga.	Biotite granite-gneiss.	do	Fine grained.	Orthoclase, microcline, oligoclase, quartz, biotite, and muscovite.	Apatite, zircon, and magnetite.
70	do	do	do	do	do	do
71	Ashland, Wis.	Olivine norite	Dark gray.	Medium grained.	Plagioclase, pyroxene, and olivine.	Magnetite and biotite.
72	do	Quartz biotite monzonite.	Pink	Coarse grained.	Plagioclase, microcline, quartz, and biotite.	Titanite, magnetite, hornblende, and apatite.
73	Amberg, Wis.	Biotite granite	Dark gray.	Medium grained.	Orthoclase and microcline, quartz, and biotite.	Zircon
74	do	do	Reddish brown.	do	do	
75	do	do	Reddish gray.	Coarse grained.	do	
76	Montello, Wis.	Hornblende granite.	Reddish brown.	Fine grained.	Orthoclase and microcline, quartz, and hornblende.	Magnetite, chlorite, and calcite.
77	Wausau, Wis.	Biotite hornblende.	do	Medium grained.	Orthoclase and microcline, plagioclase, quartz, hornblende, and biotite.	Magnetite
78	Cold Spring, Minn.	Hornblende biotite granite.	Pink	Coarse grained.	Microcline and orthoclase, plagioclase, quartz, hornblende, and biotite.	Magnetite and apatite.

See footnotes at end of table.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength. The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (or explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Secondary chlorite and epidote.	0.56	*a*+, *b*+, *c*+, *d*−, *e*+.	Monumental	C	Good contrast between polished and hammered finishes.
	.61	*a*, *b*, *c*, *d*−, *e*+	do	C	
Secondary chlorite	.31	*a*, *b*−, *c*−, *d*+, *e*−.	Architectural and constructional.[36]	C	
Plagioclase, sometimes altered to calcite and sericite.	1.42	*a*−, *b*, *c*, *d*−, *e*+	Constructional	B	Occasional intergrowths between quartz and feldspars.
Secondary chlorite, epidote, kaolin, and white mica.		*a*, *b*−, *c*−	Curbing, paving, and crushed stone.	C	The banded structure is caused by a partial segregation of the biotite, quartz, and feldspars into layers.
do		*a*, *b*−, *c*	do	C	Do.
Slight alteration of plagioclase to calcite or sericite.	1.76	*a*−, *b*+, *c*−, *d*−, *e*−.	Architectural and monumental.	B	
Plagioclase frequently weathered to calcite and sericite at center of crystals.	1.24	*a*−, *b*+, *c*+, *d*+, *e*−.	Architectural and constructional.	B	
Evidence of alteration in some of the feldspars.	0.31	*a*+, *b*−, *c*−, *d*−	Architectural and monumental.	D	Surface granulation of the components and bending of the longer feldspars are in evidence. (1896.)
	.67	do	do	D	Mica shows some degree of alignment and segregation into sheets.
Some feldspars partly altered to epidote.	.32	*a*−, *b*−, *c*−, *d*−	do	D	A striking combination of colors. Quartz and feldspars show surface granulation.
Secondary hematite	.63	*a*+, *b*, *c*−, *d*−, *e*−.	Monumental[37]	D	The color is caused by hematite, which occurs mainly along the cleavage planes of the feldspars. (1880.)
Magnetite partly altered to hematite.	0.38	*a*+, *b*−, *c*−, *d*−, *e*+.	do	D	Both quartz and feldspar show surface granulation. (1895.)
	.68	*a*−, *b*+, *c*−, *d*−.	Architectural and monumental.[38]	E & F	(1913.)

TABLE 1.—*Description of granites*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of
b=porosity, *c*=absorption, *d*=water transmission rate, *e*=deterioration in the soaking and drying test.
"—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
79	Isle, Minn.	Biotite granite	Gray	Medium to coarse.	White feldspar, quartz, and biotite.	
80	Rockville, Minn.	do	Pinkish gray.	Coarse grained.	Orthoclase and microcline, quartz, plagioclase, and biotite.	Hornblende, Magnetite, apatite, and sphene.
81	do	do	do	do	do	do
82	Morton, Minn.	Biotite granite-gneiss.	Pink and black.	Medium to coarse.	Orthoclase, plagioclase, quartz, microcline, and biotite.	Hornblende and magnetite.
83	do	do	do	do	do	do
84	do	do	do	do	do	do
85	Ortonville, Minn.	Biotite granite	Dark red	Medium grained.	Orthoclase, microcline, quartz, and biotite.	
86	do	do	do	Medium to coarse grained.	do	
87	St. Cloud, Minn.	Quartz monzonite.	Bluish gray.	Fine grained	Orthoclase, plagioclase, hornblende, quartz, microcline, and biotite.	Magnetite, sphene, apatite, and zircon.
88	do	Biotite hornblende granite.	Pink	Medium to coarse.	Orthoclase and microcline, plagioclase, quartz, hornblende, and biotite.	Magnetite
89	Delano, Minn.	Quartz biotite monzonite.	do	Medium grained.	Microcline, plagioclase, quartz, and biotite.	Magnetite, apatite, and titanite.
90	Hibbing, Minn.	Quartz monzonite.	do	do	Orthoclase, plagioclase, quartz, biotite, and hornblende.	Biotite and hornblende.
91	Peekskill, N. Y.	Biotite muscovite granite.	Buff	Medium to fine grained.	Albite, oligoclase, quartz, orthoclase, biotite, and muscovite.	Magnetite, zircon, and apatite.
92	do	do	do	do	do	do
93	Picton Island, N. Y.	Biotite granite	Red	Medium grained.	Microcline and orthoclase, plagioclase, quartz, and biotite.	Hematite, magnetite, hornblende, muscovite, titanite, apatite, and pyrite.
94	Jones Point, N. Y.	Quartz monzonite.	Light gray.	do	Plagioclase, quartz, and microcline.	Titanite

See footnotes at end of table.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength, The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (or explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
	.56	*a*−, *b*+, *c*−, *d*−, *e*−.	Architectural and monumental.	E & F	(1915.)
	.74	*a*−, *b*−, *c*−, *d*−, *e*+.	do [40]	do	(1907.)
	.74	*a*−, *b*−, *c*−, *d*−, *e*−.	do	do	
	.54	*a*, *b*−, *c*−, *d*−, *e*−.	do	do	The black minerals form prominent contorted bands. (1889.)
	.74	*a*−, *b*+, *c*−, *d*−, *e*−.	do [41]	do	The black minerals form prominent contorted bands.
	.40	*a*, *b*−, *c*−, *d*, *e*−	do	do	
	.32	*a*+, *b*−, *c*−, *d*−	Monumental	do	Somewhat gneissic structure, occasional biotite "knots."
	.82	*a*, *b*−, *c*−, *d*−	do	do	The rock has a somewhat gneissic structure.
Secondary calcite		*a*+, *c*−	Architectural and monumental.[42]	do	The quartz occurs mainly in narrow seams between the feldspars. (1868.)
do		*a*−, *c*−	do [43]	do	(1870.)
Both titanite and biotite are considerably altered, the first to leucoxene and the second to chlorite.		*a*+, *b*+, *c*−, *d*−		B	Apparently some of the pores are sealed.
Biotite and hornblende somewhat altered to chlorite.	.49	*a*+, *b*−, *c*−, *d*−		E	
Secondary chlorite	.09	*a*, *b*−, *c*, *d*+, *e*−	Architectural and monumental.[44]	G	The buff color is caused by limonite stains mainly between the grains. (1892.)
do		*a*, *b*, *c*, *d*−	do	G	
		a+, *c*−	Architectural and constructional.[45]	G	
Both feldspars are slightly altered to sericite.		*a*+, *c*	Constructional	B	

TABLE 1.—*Description of granites*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of
b=porosity, *c*=absorption, *d*=water transmission rate, *e*=deterioration in the soaking and drying test.
"—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
95	St. Peters, Pa.	Gabbro	Dark gray.	Fine grained.	Plagioclase and pyroxene.	Magnetite
96	Lambertville, N. J.	do	do	do	Plagioclase, pyroxene and biotite.	Quartz
97	Port Deposit, Md.	Biotite granite gneiss.	Bluish gray.	Medium grained.	Oligoclase, orthoclase microcline, quartz, and biotite.	Apatite, zircon, titanite, allanite, garnet, tourmaline, magnetite, hornblende, epidote, chlorite, and calcite.
98	do	do	do	do	do	do
99	Potomac, Md.	Quartz chlorite schist.	Greenish gray.	Fine grained.	Quartz, chlorite, muscovite, microcline, and plagioclase.	Magnetite and epidote.
100	do	do	do	do	do	do
101	Bethesda, Md.	Biotite-muscovite granite-gneiss.	Bluish gray.	Medium grained.	Microcline, quartz, plagioclase, biotite, and muscovite.	Apatite
102	Washington, D. C.	Metadiorite	do	Fine grained.	Plagioclase, quartz, biotite, and hornblende.	
103	Rock Creek Park, Washington, D. C.	Biotite-muscovite granite.	Gray	Medium grained.	Microcline, quartz, plagioclase, biotite, epidote, and muscovite.	Orthoclase, apatite, and titanite.
104	Graniteville, Mo.	Biotite granite	Red	do	Plagioclase, microcline orthoclase, perthite, quartz, and biotite.	Biotite, chlorite, magnetite, hematite, muscovite, and fluorite.
105	Little Rock, Ark.	Syenite	Gray	Coarse grained.	Orthoclase, biotite, pyroxene, and analcite.	Titanite, magnetite, plagioclase, and apatite.
106	Marble Falls, Tex.	Biotite granite	Pink	do	Quartz, microcline, plagioclase, and biotite.	Magnetite and titanite.
107	Kingsland, Tex.	do	do	do	do	do
108	do	do	do	do	do	do
109	Mountain Cross, Colo.	Granodiorite	Bluish gray.	Fine grained.	Plagioclase, quartz, biotite, green hornblende, and orthoclase.	Titanite and magnetite.
110	Prescott, Ariz.	do	do	do	Plagioclase, quartz, microcline, and biotite.	Magnetite, apatite, augite, and titanite.
111	Columbia National Forest, Wash.	Basalt porphyry.	Nearly black.	do	High-iron glass and plagioclase.	Pyroxene and magnetite.

See footnotes at end of table.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength. The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographical analyses (or explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Very small amount of secondary magnetite and chlorite.	.....	*a*+, *b*, *c*−, *d*−.....	Monumental.....	B	
Feldspar slightly altered to sericite or kaolin.	.....	*a*+, *c*−.....	.....	B	
Secondary chlorite and calcite.	0.36	*a*, *b*, *c*−, *d*−.....	Architectural and constructional.[46]	H	(1816).
do.....	.....	*a*−, *b*, *c*+, *d*−.....	do.....	H	
Feldspars considerably altered to sericite.	.....	*a*−, *c*+.....	Constructional.....	B	The microcline forms large eyes about which lamellar minerals are bent.
do.....	.....	*a*, *c*.....	do.....	B	Do.
Plagioclase extensively altered to sericite; biotite partly to chlorite.	.....	*a*−, *c*−.....	Monumental.....	B	Material no longer in production.
Secondary zoisite.....	.....	do.....	Rubble masonry.....	I	
Microcline and plagioclase slightly weathered to sericite.	.....	*a*−, *b*+, *c*+.....	do.....	B	Material has a gneissic structure.
Secondary kaolin and magnetite.	.....	*a*+, *b*−, *c*−.....	Architectural and monumental.[47]	J	
Orthoclase extensively altered to apatite and analcite.	.....	*a*−, *b*+, *c*+.....	Architectural and constructional.	B	Stone has a poikilitic texture.
Microcline slightly altered to sericite. Biotite somewhat chloritized.	.56	*a*, *b*−, *c*−, *d*−, *e*−.	do.[48].....	B	
.....	.82	*a*−, *b*, *c*−, *d*+, *e*−.	do.....	B	
.....	.63	*a*−, *b*, *c*−, *d*−.....	do.....	B	
Plagioclase slightly altered to calcite, fibrous quartz, and sericite.	.....	*a*+, *b*−, *c*−.....	.....	B	The material has a perceptible gneissic structure.
Marked alteration of plagioclase to calcite, sericite, and epidote.	.....	*a*−, *c*.....	.....	B	
.....	.85	*a*+, *b*−, *c*−, *d*−.....	.....	B	Porphyritic feldspars usually less than ½ in. in size.

TABLE 1.—*Description of granites*

[Calcium carbonate in column 9 was determined from that part of the sample which was soluble in 1 part of b=porosity, c=absorption, d=water transmission rate, e=deterioration in the soaking and drying test. "—" 10 percent or more below average. When no sign follows a letter, the results were within 10 percent

Serial No.	Source	Classification	Color	Texture	Mineral constituents in descending order of abundance	
					Major	Minor
(1)	(2)	(3)	(4)	(5)	(6)	(7)
112	Hillsboro, Oreg.	Augite-diorite	Greenish gray.	Medium grained.	Plagioclase (andesine) and pyroxene.	Magnetite
113	San Francisco, Calif.	Biotite-muscovite granite.	Light gray.	Medium to fine grained.	Microcline, quartz, plagioclase, biotite, and muscovite.	
114	do	do	do	do	do	
115	Alhambra, Calif.	Hypersthene gabbro.	Dark gray.	Fine grained.	Plagioclase, pyroxene, and hornblende.	Magnetite and biotite.
116	do	do	do	do	do	do

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K. Tarr and Neuman, A Study of the Effects of Heat on the Missouri Granites, Engineering Experiment Station Series 14, University of Missouri (1914).

[1] New York Customhouse; Chicago and Northwestern Railway Station, Chicago, Ill.; State Savings Bank, Lansing, Mich.; West Street Building and United States Realty Building, New York, N. Y.; piers for Hell Gate Bridge and large columns in St. John's Cathedral in New York City.
[2] Post Office Department Building (old), Washington, D. C.; Masonic Temple, Philadelphia, Pa.; Savings Bank, Wilmington, Del.; Board of Trade Building, Chicago, Ill.; Post Office and Customhouse, Brooklyn, N. Y.; Manhattan Bank, New York City; General Wood's Monument, Troy, N. Y.; part of New York Customhouse.
[3] United States Mint, Philadelphia, Pa.; basement of New York Customhouse; Brooklyn anchorage of Manhattan bridge; basement of House Office Building (new), Washington, D. C.; Lafayette High School, Brooklyn, N. Y.; interior of St. John's Cathedral, New York City.
[4] Customhouse, Buffalo, N. Y.; Customhouse, Fall River, Mass.; Western Savings Bank, Philadelphia, Pa.; National Bank of Commerce, Kansas City, Mo.; Bellevue Hospital, New York City; Winter Mausoleum, Pittsburgh, Pa.; Kress Mausoleum, Allentown, Pa.
[5] Post Office, Albany, N. Y.; Bates Building, Philadelphia, Pa.; part of the Saratoga Monument, Saratoga, N. Y.
[6] City Hall, Portland, Maine; C & N Railway and Field Annex buildings, Chicago, Ill.; Union Trust Building, South Bend, Ind.; Exchange National Bank, Tulsa, Okla.; Penn Mutual Life Building, Philadelphia, Pa.; General Grant's Tomb, New York City; Hahnemann Monument, Washington, D. C.
[7] Post Office, Lowell, Mass.; Museum of Fine Arts, Boston, Mass.; Ninth Regiment Armory, New York City; National Security Bank, Los Angeles, Calif.; Stevens Hotel (base), Chicago, Ill.; Insurance Company of North America building, Philadelphia, Pa.; South Office Building of the State Capitol, Harrisburg, Pa.; New York State Office Building, Albany, N. Y.; Pioneer Monument, Harrodsburg, Ky.

and their uses—Continued

HCl and 9 parts of water. Physical properties in column 10 are denoted as follows: *a*=compressive strength. The signs "+" following letters denote that the results of tests were 10 percent or more above average and of average. Test values for the various samples are given in table 3 and figure 8.]

Mineral alterations	Amount of calcium carbonate	Physical characteristics	Main uses	Authority for petrographic analyses (or explanation of A, B, etc., see references at end of table)	Remarks and year production started
(8)	(9)	(10)	(11)	(12)	(13)
	Percent				
Plagioclase considerably altered to albite, calcite and sericite; pyroxene to uralite and antigorite.	5.20	*a*+, *b*+, *c*-		B	
Plagioclase sometimes altered to sericite and calcite.	------	*a*-, *b*+, *c*+	Architectural and constructional.	B	Material has a slight gneissic structure.
------	------	------do------	------do------	B	
Pyroxene (hypersthene) slightly altered to hornblende and chlorite.	------	*a*+, *b*-, *c*-, *d*-	Monumental	B	
------	1.51	------do------	------do------	B	

[1] Post Office buildings at Indianapolis, Ind. (base), Cleveland, Ohio, Milwaukee, Wis., and Jersey City, N. J.; New York City Public Library (base); St. Louis Public Library; United States Mint, Philadelphia, Pa.

[2] Post Office buildings at Buffalo, N. Y., and Hartford, Conn.; Standard Oil Building, New York City.

[10] Lyman Allyn Museum, New London, Conn.; Christian Science Publishing Building, Boston, Mass.; Douglas Memorial Hall, Washington, D. C.; United States Post Office Building, Cleveland, Ohio; Liberty Mutual Insurance Building, Boston, Mass.; Southern New England Telephone Building, New Haven, Conn. All of these were erected between 1930 and 1938.

[11] Christian Science Church, Boston, Mass.; Lehigh County Court House, Allentown, Pa.; Medico-Chirugical Hospital, Philadelphia, Pa.; Post Offices at Dayton, Ohio, Lincoln, Nebr., Staten Island, N. Y., and Newark, N. J.; War Memorial, Indianapolis, Ind.; Gulf Building, Pittsburgh, Pa.; First National Bank, New York City; Rackham Memorial, Ann Arbor, Mich.

[12] First National Bank, Chicago, Ill.; First National Bank, Cincinnati, Ohio; Wisconsin Telephone Building, Milwaukee, Wis.; Franklin Savings Bank, New York City; City Hall, Lowell, Mass.; State Library, Concord, N. H.; Masonic Memorial Temple, Alexandria, Va.

[13] Union Arcade Building, Pittsburgh, Pa.; Fidelity Mutual Life Insurance Building, Philadelphia, Pa.; Post Office (polished columns), Washington, D. C.; Dime Savings Bank (exterior columns), Detroit, Mich.; Missouri State Capitol (interior columns), Jefferson City; First National Bank, Bridgeport, Conn. (exterior columns); Baxter Building, Portland, Maine.

[14] Pier No. 4 of Haverhill, Mass., bridge; trimmings of Majestic Theater, Chicago, Ill.; Ferguson Mausoleum, Kensico Cemetery, New York, N. Y.

[15] Pylon Monuments of Manhattan Bridge, Brooklyn Plaza, N. Y.; L. C. Smith Mausoleum, Syracuse N. Y.; C. W. Post Mausoleum, Battle Creek, Mich.

[16] War Memorial, Springfield, Mass. (1933); Philip B. Armour Monument, Chicago, Ill. (1918); Statue of Youth, Barre, Vt. (1923); Avery Hopwood Monument, Cleveland, Ohio (1928); Navy and Marine Memorial, Newport, R. I. (1938); Richard Bartholdt Monument, St. Louis, Mo. (1936).

[17] Iowa Soldiers' Monument, Chattanooga, Tenn.; Pennsylvania Soldiers' Monument, York, Pa.; Hearn Monument, Woodlawn, N. Y.; Rouse Mausoleum, Winchester, Va.; Soldiers and Sailors Monument, Angola, Ind.

[18] Prison-ship Martyrs' Monument in Fort Greene Park, Brooklyn, N. Y.

[19] Northwestern Mutual Life Building, Milwaukee, Wis.; Museum of Fine Arts, Minneapolis, Minn.; Soldiers' and Sailors' Memorial, Wichita, Kans.; Bridgeport Trust Building, Bridgeport, Conn.

[20] Pennsylvania State Capitol, Harrisburg, Pa.; Cook County Court House, Chicago, Ill.; City Hall, Cleveland, Ohio; Mercantile Trust Building, St. Louis, Mo.; Kentucky State Capitol (base and 36 polished columns), Frankfort, Ky.; Navy Memorial, Vicksburg, Miss.; Memorial Arch, Port Huron, Mich.

[21] Columbia University, New York City (16 polished columns); General Gomez monument in Cuba; Die in Bennington monument, Bennington, Vt.; Bank of Montreal (34 columns); W. C. T. U. fountain, Orange, Mass. (columns and die).

(Footnotes continued on p. 180)

(Footnotes continued from p. 179)

[32] Post Office, Troy, N. Y.; Diamond Bank, Pittsburgh, Pa.; McFadden Building, Chicago, Ill.; Royal Baking Powder Building, New York City.

[33] First National Bank Building, New York City (1932).

[34] Finance Building, State Capitol, Harrisburg, Pa.; New Jersey State Office Building, Trenton, N. J.; Massachusetts State House (wings), Boston, Mass.; Fidelity Mutual Life Building, Philadelphia, Pa.; Masonic Temple, St. Louis, Mo.; New Orleans Post Office (base and approaches) New Orleans, La.; County Court House, Hartford, Conn.

[35] Hanover National Bank, New York City; Public Library, Boston, Mass.; Public Library, Columbus, Ohio; National Museum (base), Washington, D. C.; Chamber of Commerce, Rochester, N. Y.; Customhouse, Wilmington, N. C.; Commercial National Bank, Chicago, Ill.

[36] Customhouse, Boston; Masonic Temple, Chicago, Ill.; Union National Bank, Pittsburgh, Pa.

[37] Hamilton Club, Chicago, Ill.; Barnes Building, Wichita, Kans.; National State Bank, Newark, N. J.; Post Office, Elkhart, Ind.; Syndicate Building, Minneapolis, Minn.; Brooks Bros. Building, New York City.

[38] City Deposit Bank, Pittsburgh, Pa.; Soldiers' and Sailors' Memorial, Syracuse, N. Y.; Alexander Hamilton Memorial, Grant Park, Chicago, Ill.

[39] South Station, Boston, Mass.; Bessemer Building, Pittsburgh, Pa.; Newberry Library, Chicago, Ill.; Battle Monument (polished columns), West Point, N. Y.; McMillan fountain, Washington, D. C.; Sheridan Monument (base), New York City; Hooker Monument (State House grounds), Boston, Mass.

[40] Telephone Building, New York City; South Terminal Station, Boston, Mass.; Grosvenor Building, Providence, R. I.; New England Telephone Building, Hartford, Conn.; Philadelphia Savings Fund Society, Philadelphia, Pa.; Provident Savings Bank, Baltimore, Md.; Rochester Savings Bank, Rochester, N. Y.; Fulton Building, Pittsburgh, Pa.; Dixie Terminal Building, Cincinnati, Ohio; Whitney Central Bank, New Orleans, La.

[41] Union Trust Co. (1906) and Arlington Bridge (1917), Washington, D. C.; Guilford County Courthouse (1917), Greensboro, N. C.; State Office Building (1938), Raleigh, N. C.; United States Depository (1936), Fort Knox, Ky.; Wright Brothers Memorial (1932), Kitty Hawk, N. C.; Rose Hill Mausoleum (1925), Rose Hill Cemetery, Chicago, Ill.

[42] Raleigh, N. C., Post Office (1875) and addition to same (1937); Governor Endicott Monument and Statue (1937), Boston, Mass.; Antietam Creek Bridge (1936) near Hagerstown, Md.

[43] Fidelity Trust Building, Philadelphia, Pa.; Blackstone Hotel, Chicago, Ill.; Butler University, Indianapolis, Ind.; President Buchanan Memorial, Washington, D. C.; Morris Mausoleum, Cleveland, Ohio; Sutherland Exhedra, Greenwood Cemetery, Brooklyn, N. Y.; World War Memorial, Des Moines, Iowa; Lewis and Clark Memorial, Charlottesville, Va.

[44] Field office, George Washington Bridge, Fort Lee, N. J.; High School, Asheville, N. C.; Hilton Parkway bridges, Baltimore, Md.; Post Office, Wilmington, Del.; Howard University power plant and Chemistry Building, Washington, D. C.; Loyola University, Chicago, Ill.; Beardsley Monument, Indianapolis, Ind.

[45] United States Post Office and Court House buildings at Wilmington, N. C., and Asheville, N. C.; Post Office buildings at Charleston, S. C., Durham, N. C., Chillicothe, Ohio, Traverse City, Mich., Florence, S. C., and Charlottesville, Va.; Hussey Building, Pittsburgh, Pa.; New Land and Trust Building, Philadelphia, Pa.; United States Customhouse, Baltimore, Md.; Empire Bank Building, Clarksburg, W. Va.

[46] Customhouse, Mobile, Ala.; Customhouse Extension, Denver, Colo.; Supreme Court Building, Cheyenne, Wyo.; Veterans' Administration Facility, Columbia, S. C.; Central Office "U" Building, Detroit, Mich.; Post Office, Provo, Utah; Post Office Garage, Chicago, Ill.; Frankford Arsenal, Philadelphia, Pa.; State Office, Oklahoma City, Okla.; Southern Bell Telephone Building, Charleston, S. C.; P Street Bridge, Washington, D. C.

[47] General Grant Sarcophagus in Grant's Tomb, New York City; Illinois State Monuments, Andersonville, Ga.

[48] State Office Building, St. Paul, Minn.; City Hall (1927), Duluth, Minn.; Gas and Electric Building (1925), Oklahoma City, Okla.; Ford Museum (1929), Dearborn, Mich.; Merchants Mart (1929), Chicago, Ill.; Jefferson County Courthouse (1929), Birmingham, Ala.; Criminal Court Building (1928), New Orleans, La.; Cadillac Building (1926), Boston, Mass.; New York Central Station (1926), Buffalo, N. Y.; Gallinger Hospital, (1925), Washington, D. C.; Koppers Building (1927), Pittsburgh, Pa.; City Hall (1926), Columbus, Ohio; Chamber of Commerce (1935), Jacksonville, Fla.; Telephone Building (1925), Dallas, Tex.; First National Bank (1931), San Jose, Calif.; Young's Market (1922), Los Angeles, Calif.; Schroeder Hotel (1928), Milwaukee, Wis.; Post Office (1926), Des Moines, Iowa.

[49] Coliseum (1936), Fort Worth, Tex.; County Court House (1936), Oklahoma City, Okla.; Post Office (1936), Jacksonville, Ill.; Marine Hospital (1937), St. Louis, Mo.; State Capitol (1930), Baton Rouge, La.; Telephone Building, Evansville, Ind.; Price Building, Kansas City, Mo.; County Court House (1938), Jacksonville, Miss.; Capitol Club Office Building, Raleigh, N. C.

[50] The Cathedral, St. Paul, Minn.; Tribune Tower (base), Chicago, Ill.; United States Court House (1934), Foley Square, New York City; Capitol Plaza Fountain (1931), Washington, D. C.; addition to Milwaukee Post Office (1930); Post Office (1928), Duluth, Minn.; Book Cadillac Hotel, Detroit, Mich.; Justice Building, St. Louis, Mo.; New City Hall, Kansas City, Mo.; Telephone Building, Oklahoma City, Okla.

[51] Telephone Building (1936), St. Paul, Minn.; Vanderbilt Monument (1936), New York City; Esquire Theatre (1937), Chicago, Ill.; Dunning Mausoleum (1937), Fort Worth, Tex.; Post Office (1937), Los Angeles, Calif.; Telephone Building (1931), Cincinnati, Ohio; Adler Planetarium (1928), Chicago, Ill.; David Stott Building (1925), Detroit, Mich.; Mariner Tower Building (1930), Milwaukee, Wis.

[52] Post Office Building (1868), St. Paul, Minn.

[53] Post Office, St. Cloud, Minn. (1902); base of old Minneapolis Post Office (1884).

[54] St. John's Cathedral (interior), residence of Charles M. Schwab, Postal Telegraph Building, Cross Building, and Bronx Geological Gardens, New York City; Post Office, Bangor, Maine; Connecticut State Armory, Hartford, Conn.

[55] American Museum of Natural History, New York City, National Bank of Clayton, N. Y.; Maryland Museum Building, Baltimore, Md. (polished columns).

[56] Fort McHenry (part of walls), Baltimore harbor, Md.; St. Dominick Church, Washington, D. C.

[57] Washington University, St. Louis, Mo.; State House, Springfield, Ill.; Paxton Building, Omaha, Nebr.; Marshall Field Building, Chicago, Ill.; Whitney Building, New Orleans, La.; Art Museum, Cincinnati, Ohio; Keith and Perry Building, Kansas City, Mo.; Central Savings Bank, Baltimore, Md.; Society for Savings, Cleveland, Ohio; F. E. Knowles and Company Building, San Francisco, Calif.; Thos. Allen Monument, Pittsfield, Mass.

[58] Texas State Capitol, Austin, Tex.; and Fort Worth Court House, Fort Worth,, Tex.

The most abundant mineral in granite is generally feldspar, which is of two main types—namely, potash feldspar, a silicate of potash and alumina; and soda-lime feldspar (plagioclase), a silicate of sodium, aluminum, and calcium. Common species of the first type are orthoclase and microcline and of the second type albite, oligoclase, andesine, labradorite, bytownite, and anorthite. Both types in granite usually show distinct crystal forms, and the crystal faces when exposed at the surface cause the sparkling effect which is often pronounced in coarse-grained granite. The color of plagioclase is usually white or gray, while the potash feldspar occurs in various colors, such as gray, pink, buff, or green.

Quartz (SiO_2) in granite is sometimes nearly clear but more often clouded by inclusions of dark particles, when it is called smoky quartz. It can be identified by its vitreous luster, irregular fracture, and greater hardness than feldspar. Quartz is probably the most inert and durable mineral in granite.

In most granites mica ranks third in abundance. Black mica (biotite) is more common than white mica (muscovite), but often both species occur in the same rock. Mica is a relatively soft mineral easily identified by the fact that it can be separated into thin sheets which are flexible.

The black basic minerals, hornblende and pyroxene, occur as prominent constituents in some granites. They can be distinguished from biotite by their greater hardness and the fact that they have no definite cleavage. These minerals are complex silicates of magnesium and iron. Several species of pyroxene, such as diopside, augite, aegirite, and riebeckite, are found in granite.

A large number of other minerals occur in granite, but generally they make up a very small part of the rock. For information on them the reader may consult a textbook on mineralogy.

"Mineral alterations" (column 8 of table 1) refer to those changes which have occurred since the original minerals formed. Most alterations are attributed to weathering, and they are confined mainly to rock near the earth's surface or near open joints. Common alterations are as follows: Feldspar to kaolin or mica; biotite to chlorite; and plagioclase, in part, to calcite.

Partial chemical analyses were made on 80 samples to determine the amounts of calcite and pyrite. The results for calcite only are given because the sulfide was uniformly low and for 71 samples it was below 0.02 percent.

Several specific examples of structural use are cited in table 1, along with the dates of construction when known. For those granites which have been used extensively, only a few representative examples in different sections of the country are referred to.

IV. PHYSICAL TESTS

1. COMPRESSIVE STRENGTH

The specimens (described in section II) were tested on a 300,000-lb hydraulic machine with a spherical adjustment block above the specimen. Disks of hardened steel 2.5 in. in diameter by 0.5 in. in thickness were placed between the ends of the specimens and the machine plates, and as these became roughened they were resurfaced. One

thickness of thin blotting paper was placed between the plates and the ends of the specimen in order to assure a satisfactory distribution of the load. The load was applied at a rate of 100 lb/in.^{2} per second. Crushing usually occurred suddenly with very little chipping of the corners before the final break.

Some producers have raised the point that very coarse-grained materials would give higher compressive strength if larger specimens were used. For this reason comparative tests were made on two sizes of specimens from each of five different granites selected from the coarsest grained materials of the series. The results of these comparisons are given in table 2.

TABLE 2.—*Effect of size of test specimen on strength of granite*

Serial No.	Size of specimen		Number of tests	Compressive strength [a]			Variation [b]
	Diameter	Height		Maximum	Minimum	Average	
	in.	*in.*		*lb/in.*^{2}	*lb/in.*^{2}	*lb/in.*^{2}	*Percent*
12	2.1	2.5	2	21,300	17,900	19,600	
12a [c]	3.0	3.5	3	26,100	25,300	25,700	+31
13	2.1	2.1	6	29,300	23,800	26,700	
13	2.5	2.5	6	25,700	22,600	24,300	−9
36	2.1	2.5	2	20,400	18,900	19,600	
36	2.5	3.0	4	25,700	22,000	23,800	+21
81	2.1	2.5	2	23,300	17,300	20,300	
81	2.5	3.0	4	23,200	21,200	22,200	+9
107	2.1	2.5	2	21,700	16,200	19,000	
107	2.5	3.0	4	23,400	20,400	22,100	+16

[a] All specimens tested dry and loaded perpendicular to rift.

[b] Computed by the formula $\frac{(C_2-C_1)\,100}{C_1}$, in which C_1 and C_2 are, respectively, the average compressive strengths of the small and large sizes of specimens.

[c] Serial No. 12a was from the same quarry as No. 12 but taken at a later date.

The specimens showing greatest difference in strength between the two sizes were cut from two different samples from the same quarry (12 and 12a), and hence the comparisons may not be justified in this case. If the average values are weighted in accordance with the number of tests for each size, the weighted mean for all the larger specimens is about 4 percent higher than that for all the smaller specimens. If a similar computation is made omitting the values for 12 and 12a, the results indicate no appreciable difference.

Results of compression tests on cylindrical specimens 2.1 by 2.5 in. are given in table 3 for 114 samples of granite. The average strength for all of the granites in the dry condition was 24,500 lb/in.^{2} and the highest recorded was 60,000 lb/in.^{2} Occasional strengths below 10,000 lb/in.^{2} were obtained for samples apparently taken near the surface of the ground. Some granites appear to be stronger when loaded perpendicular to the rift, but the grand averages of all values for each of the two directions of loading were practically the same. Many of the samples showed a reduction in strength for the wet condition, and the average of all tests on wet specimens was about 12 percent lower than on dry specimens.

TABLE 3.—*Average values obtained for compressive strength, absorption, density porosity and weight per cubic foot of various granites, by States*

Serial No.	Compressive strength perpendicular (*A*) and parallel (*B*) to the rift				Absorption by weight for immersion periods of			Density		Porosity *	Computed weight per cu. ft.
	Specimens dry		Specimens wet		48 hr	14 days	1 yr	Bulk	True		
	A	*B*	*A*	*B*							
					MAINE						
	lb/in.²	*lb/in.²*	*lb/in.²*	*lb/in.²*	%	%	%			%	*lb.*
1	27,600	29,200	26,500	31,800	0.23	0.25	0.28	2.624	2.657	1.24	164
2	34,800	37,000	24,100	33,400	.03	.03	.04	2.964	2.976	.40	185
3	16,900	21,200	14,400	15,000	.26	.32	.38	2.619	2.658	1.47	163
4	19,400	26,300	27,300	25,100	.20	.25	.31	2.628	2.655	1.02	164
5	28,000	29,700	27,800	33,800	.15	.18	.21	2.637	2.662	.94	164
6	16,900	-------	-------	-------	-------	.21	-------	2.65	-------	-------	165
7	33,000	26,400	15,600	16,700	.21	.28	.35	2.605	2.645	1.51	162
8	38,800	35,300	33,000	31,600	.23	.28	.34	2.611	2.642	1.14	163
9	22,800	23,200	23,000	21,900	.23	.31	.47	2.602	2.644	2.00	162
10	22,900	23,600	17,000	17,300	.22	.25	.29	2.660	2.682	.83	166
11	29,600	21,100	24,400	21,000	.31	.36	.39	2.623	2.658	1.32	164
12	19,600	21,300	22,800	19,900	.22	.29	.40	2.610	2.645	1.36	163
13	26,700	28,500	22,600	24,200	.32	.36	.41	2.623	2.659	1.36	164
14	18,700	-------	-------	-------	-------	.34	-------	2.63	-------	-------	164
15	19,400	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
16	26,000	20,600	20,300	21,400	.32	.38	.44	2.602	2.641	1.48	162
					NEW HAMPSHIRE						
17	24,900	21,100	19,200	15,600	0.29	0.32	0.36	2.630	2.668	1.42	164
18	19,800	20,900	19,900	17,600	.23	.27	.34	2.613	2.646	1.25	163
19	24,200	19,200	19,800	15,000	.20	.22	.28	2.623	2.648	.94	164
20	22,800	22,600	15,500	14,900	.34	.37	.41	2.626	2.663	1.39	164
21	19,000	16,800	17,600	17,000	.29	.34	.39	2.626	2.660	1.28	164
22	23,300	20,000	-------	-------	-------	.26	-------	2.65	-------	-------	166
23	33,100	-------	-------	-------	-------	.34	-------	2.65	-------	-------	166
24	22,200	25,600	22,500	24,400	.25	.28	.32	2.635	2.667	1.20	164
					VERMONT						
25	15,600	14,400	14,200	16,900	0.20	0.22	0.26	2.660	2.686	0.97	166
26	27,000	-------	22,300	-------	-------	.27	-------	2.64	-------	-------	165
27	18,600	20,600	15,500	14,200	.26	.28	.34	2.646	2.678	1.27	165
28	29,800	29,100	24,600	25,500	.27	.31	.41	2.628	2.675	1.76	164
29	13,800	13,000	14,200	14,600	.24	.27	.30	2.661	2.695	1.26	166
30	26,400	35,500	28,000	27,300	.14	.15	.16	2.644	2.672	1.05	165
31	22,300	19,700	20,400	17,300	0 .24	.28	-------	2.67	-------	-------	167
32	26,700	-------	20,600	-------	-------	.29	-------	2.63	-------	-------	164
					MASSACHUSETTS						
33	18,600	24,400	23,900	22,200	0.26	0.29	0.34	2.639	2.678	1.46	165
34	24,600	24,200	23,700	25,800	.25	.29	.34	2.636	2.674	1.53	164
35	23,600	30,000	23,400	22,700	.21	.24	.28	2.635	2.677	1.57	164
36	19,800	21,800	24,300	21,000	.26	.30	.34	2.644	2.678	1.27	165
37	24,800	29,600	20,600	18,800	.32	.36	.43	2.617	2.658	1.51	163
38	23,800	25,400	20,000	21,300	.32	.36	.44	2.616	2.659	1.62	163
39	30,800	24,600	23,400	22,500	.26	.27	.32	2.664	2.700	1.33	166
40	28,000	25,900	22,600	24,200	.22	.25	.28	2.620	2.657	1.39	163
41	-------	-------	-------	-------	.15	.20	-------	2.62	-------	-------	163
42	-------	-------	-------	-------	.11	.13	-------	2.63	-------	-------	164
					CONNECTICUT						
43	16,800	19,800	21,300	22,000	0.21	0.23	0.28	2.622	2.651	1.09	163
44	22,400	23,800	22,600	21,600	.19	.22	.26	2.625	2.657	1.21	164
45	30,800	26,800	25,600	30,000	.22	.26	.34	2.626	2.666	1.50	164
46	24,200	32,400	15,900	21,000	.31	.34	.48	2.617	2.663	1.73	163
47	23,800	26,900	22,000	24,700	.27	.31	.40	2.614	2.646	1.21	163
48	20,000	24,000	14,200	14,400	.18	.20	.28	2.634	2.662	1.05	164

TABLE 3.—*Average values obtained for compressive strength, absorption, density, porosity and weight per cubic foot of various granites, by States*—Continued

Serial No.	Compressive strength perpendicular (*A*) and parallel (*B*) to the rift				Absorption by weight for immersion periods of			Density		Porosity [a]	Computed weight per cu. ft.
	Specimens dry		Specimens wet		48 hr	14 days	1 yr	Bulk	True		
	A	*B*	*A*	*B*							
NORTH CAROLINA											
	*lb/in.*²	*lb/in.*²	*lb/in.*²	*lb/in.*²	%	%	%			%	*lb.*
49	47,400	35,100	37,600	24,800	0.06	0.10	0.13	2.885	2.930	1.54	180
50	41,200	.....	32,400	.....	.09	.11	.15	2.861	2.890	1.00	179
51	22,600	21,900	17,300	18,400	.20	.24	.31	2.628	2.656	1.05	164
52	17,500	17,200	16,600	12,800	.17	.20	.28	2.654	2.688	1.26	165
53	18,600	20,400	17,400	15,500	.25	.28	.31	2.631	2.657	.98	164
54	14,200	17,300	13,000	9,000	.29	.31	.36	2.627	2.665	1.43	164
55	39,700	32,200	30,000	34,200	.16	.17	.23	2.614	2.643	1.10	163
56	34,800	34,400	33,500	32,700	.16	.21	.23	2.614	2.639	.95	164
57	28,600	25,100	23,000	26,400	.18	.21	.25	2.611	2.640	1.10	162
58	30,600	23,100	26,000	24,600	.19	.21	.27	2.614	2.648	1.28	163
SOUTH CAROLINA											
59	22,600	19,600	22,800	20,800	0.25	0.28	0.40	2.637	2.673	1.35	164
60	25,100	.....	.....	.....	.29	.32	.....	2.64	.....	.....	165
61	33,200	34,200	27,800	26,000	.26	.29	.38	2.632	2.692	2.23	164
62	27,300	23,500	27,800	28,300	.22	.23	.30	2.636	2.670	1.27	164
63	30,700	.....	.....	.....	.....	.32	.....	2.63	.....	.....	164
64	29,400	24,600	23,800	20,400	.24	.30	.36	2.663	2.695	1.19	166
GEORGIA											
65	29,400	26,500	24,900	23,600	0.30	0.33	0.40	2.622	2.662	1.50	163
66	26,300	21,400	25,400	22,500	.24	.26	.30	2.628	2.663	1.31	164
67	22,600	25,000	18,500	17,800	.19	.21	.24	2.631	2.657	.98	164
68	21,000	23,400	15,500	16,500	.26	.29	.36	2.675	2.710	1.29	166
69	27,800	24,000	20,800	19,800	.22	.24	.....	2.623	2.648	.96	162
70	22,700	25,600	20,800	22,400	.19	.22	.....	2.634	2.660	.98	164
WISCONSIN											
71	13,800	14,400	17,400	16,800	0.05	0.05	0.06	2.911	2.954	1.45	181
72	16,900	20,200	14,700	15,000	.30	.37	.46	2.662	2.710	1.77	166
73	27,900	32,100	30,800	26,200	.07	.08	.09	2.671	2.687	.59	166
74	37,000	30,600	31,700	31,200	.12	.15	.15	2.624	2.649	.94	164
75	22,700	18,900	16,800	15,400	.10	.12	.14	2.628	2.652	.91	164
76	29,500	42,400	27,800	34,200	.05	.05	.06	2.629	2.662	1.24	164
77	31,200	30,400	25,800	28,200	.08	.08	.10	2.616	2.640	.91	163
MINNESOTA											
78	19,000	.....	12,000	13,000	0.12	0.17	0.22	2.681	2.754	2.65	167
79	18,300	17,400	16,800	17,200	.11	.15	.22	2.637	2.680	1.60	164
80	19,000	20,000	17,400	16,000	.10	.12	.15	2.684	2.710	.96	167
81	20,300	21,400	14,400	15,000	.10	.....	.15	2.679	2.700	.78	167
82	21,800	30,700	26,000	28,600	.19	.23	.24	2.659	2.688	1.08	166
83	17,000	16,300	20,600	17,100	.07	.10	.12	2.688	2.732	1.83	167
84	21,600	35,400	26,700	26,700	.19	.22	.26	2.666	2.684	.67	166
85	29,400	24,200	23,100	20,400	.13	.14	.18	2.626	2.651	.94	164
86	22,000	21,700	18,700	21,400	.12	.13	.17	2.673	2.696	.96	166
87	26,900	.....	.....	.....	.15	.19	.....	2.70	.....	.....	168
88	20,700	.....	.....	.....	.15	.....	.....	2.62	.....	.....	163
89	29,600	26,800	19,300	23,300	.11	.12	.....	2.656	2.694	1.41	165
90	37,200	39,800	30,200	30,600	.19	.22	.28	2.624	2.650	.98	164

TABLE 3.—*Average values obtained for compressive strength, absorption, density, porosity and weight per cubic foot of various granites, by States*—Continued

Serial No.	Compressive strength perpendicular (*A*) and parallel (*B*) to the rift				Absorption by weight for immersion periods of			Density		Porosity [a]	Computed weight per cu. ft.
	Specime dry		Specimens wet		48 hr	14 days	1 yr	Bulk	True		
	A	*B*	*A*	*B*							
MISCELLANEOUS											
	lb/in.²	*lb/in.²*	*lb/in.²*	*lb/in.²*	%	%	%			%	*lb.*
91	22,200	22,000	21,000	20,000	0.22	0.24	0.27	2.609	2.636	1.02	162
92	24,400	21,400	20,200	20,400	.24	.24	.30	2.607	2.639	1.22	163
93	24,800	-------	30,400	-------	-------	.13	-------	2.64	-------	-------	165
94	30,200	-------	-------	-------	.21	-------	-------	2.63	-------	-------	164
95	44,100	35,000	40,000	41,000	.16	.17	.20	3.041	3.080	1.27	190
96	45,200	-------	41,500	-------	.10	.10	-------	2.96	-------	-------	185
97	23,700	24,900	22,000	17,600	.15	.16	.20	2.693	2.725	1.17	168
98	20,600	18,700	17,500	20,100	.28	.30	-------	2.682	2.716	1.25	167
99	20,000	-------	-------	-------	-------	.63	-------	2.74	-------	-------	171
100	21,800	-------	-------	-------	-------	.21	-------	2.80	-------	-------	175
101	20,500	-------	16,900	-------	-------	.14	-------	2.70	-------	-------	169
102	18,000	-------	-------	-------	.08	-------	-------	-------	-------	-------	-------
103	7,700	-------	-------	-------	-------	.40	-------	2.692	2.738	1.68	168
104	29,700	-------	28,100	-------	.13	.16	-------	2.609	2.639	1.14	162
105	17,800	-------	-------	-------	-------	.89	-------	2.510	2.609	3.84	157
106	22,500	25,000	21,600	23,400	.14	.18	.27	2.621	2.648	1.02	163
107	19,000	20,500	18,400	15,400	.15	.18	.23	2.642	2.674	1.20	165
108	16,900	17,400	20,000	18,600	.15	.17	.22	2.646	2.680	1.27	165
109	30,400	-------	-------	-------	-------	.14	-------	2.753	2.775	.79	172
110	15,300	-------	-------	-------	-------	.26	-------	2.66	-------	-------	166
111	-------	-------	43,800	56,800	.02	.03	-------	2.573	2.599	1.00	160
112	33,200	36,300	29,400	26,000	.22	.26	-------	2.782	2.842	2.10	174
113	21,600	-------	19,500	-------	.29	-------	-------	2.639	2.682	1.62	165
114	18,200	-------	16,400	-------	.58	-------	-------	2.631	2.696	2.40	164
115	53,800	54,400	32,200	41,800	.04	.06	.07	3.001	3.024	.76	187
116	34,400	35,300	30,600	31,200	.07	.08	.09	2.880	2.910	1.03	180

[a] Total pore space including that of closed pores.

One feature worthy of note regarding some of the granites is the rather large variation in strength shown by specimens cut from a single sample. The deviations of the individual values from the average of each pair of like specimens similarly tested averaged 7 percent for serial numbers 1 to 58, but the deviation of three of these was more than 20 percent.

The following values, showing the range in compressive strength recorded for other masonry materials, are given for comparison:

Material	Compressive strength
	lb/in.²
Marble	8,000 to 27,000
Limestone	2,600 to 28,000
Sandstone	5,000 to 20,000
Quartzite	16,000 to 45,000
Serpentine	11,000 to 28,000
Cast stone	1,600 to 21,800
Brick	1,000 to 20,000

2. FLEXURAL STRENGTH

Stone in modern construction is seldom subjected to high bending stresses, and hence no systematic study of flexural strength was made on the granites. The modulus of rupture was determined on 29 specimens representing 5 samples. The specimens (4 by 12 by 1 in.) were cut, as shown in figure 1, for tests in various directions, supported on adjustable knife edges 10 in. apart, and loaded at the center. The modulus of rupture was computed from the formula

$$R=\frac{3Wl}{2bt^2},$$

in which W is the load at the middle of span required to produce rupture, l is the length of span in inches, b the width of the specimen, and t the thickness. The results are given in table 4.

Of the samples tested both perpendicular and parallel to the rift, one (12b) was stronger for the first condition and the other (89) stronger for the second. This seems to agree with indications obtained in compression tests that there is no consistent difference in strength between the two directions.

TABLE 4.— *Results of flexure tests on granite*

Serial No.[a]	Manner of loading [b]	Number of tests	Modulus of rupture		
			Maximum	Minimum	Average
			lb/in.^{2}	*lb/in.*^{2}	*lb/in.*^{2}
12b	*A*	5	1,630	1,450	1,540
12b	*B*	5	1,520	1,380	1,430
12b	*C*	5	1,560	1,410	1,490
26	*A*	3	1,980	1,910	1,930
83	*C*	2	3,170	2,940	3,060
89	*A*	3	2,450	2,180	2,290
89	*B*	3	2,520	2,450	2,500
109	*A*	3	5,550	4,970	5,190

[a] Serial No. 12b was from the same quarry as No. 12.
[b] For test *A* the specimens were prepared as shown in figure 1, at 3; for test *B* as at 5; and for test *C* as at 4.

The following values show the range in modulus of rupture recorded for other types of stone:

Material	Modulus of rupture
	lb/in.^{2}
Marble	600 to 4,000
Limestone	500 to 2,000
Slate	6,000 to 15,000
Serpentine	1,300 to 11,000
Sandstone	700 to 2,300

3. SHEARING STRENGTH

Shearing strength of granite may be of more concern to the designer of granite structures than the compressive strength, because the latter is usually far in excess of that required for all ordinary conditions. The load on large blocks of granite in masonry is sometimes concentrated near the edge because of improper bedding with mortar, and spalls occur near the edges which may be due in part to high shearing stresses.

A few determinations of shearing strength on granite were made with the punching shear device shown in figure 2, and the results are given in table 5. This device consists essentially of a base plate carrying two vertical guide rods and a 2-in. plunger provided with a spherical adjustment block at the bottom and a loading table at the top. The function of the coil spring is merely to support the loading table and plunger at a convenient distance above the base plate while the specimen is placed in position. In making the tests, the load necessary to compress the spring until the plunger makes contact with the specimen is determined and subtracted from the load required to punch out a 2-in. disk.

Specimens on which these tests were made were slabs approximately 4 by 6 by 1 in. The shearing strength was computed by means of the formula

$$S=\frac{P}{2\pi t},$$

in which P is the total load minus the load required to compress the spring and t is the thickness of specimen.

The results given in table 5 range from 3,900 to 4,600 lb/in.^{2} The ratios between the shearing strength and the corresponding compressive strength of the samples are given in the last column.

TABLE 5.—*Results of shear tests on granite*

Serial No.[a]	Number of tests	Shearing strength			Ratio S/C[b]
		Maximum	Minimum	Average	
		lb/in.^{2}	*lb/in.*^{2}	*lb/in.*^{2}	
12b	2	4,800	4,400	4,600	0.23
29a	2	4,100	3,700	3,900	.29
83	3	4,700	3,900	4,600	.27
89	3	4,600	4,200	4,300	.15

[a] Serial Nos. 12b and 29a were, respectively, from the same quarries as Nos. 12 and 29.
[b] Shearing strength divided by the compressive strength.

The following results show the range in shearing strength recorded in the literature for other types of stone:

Material	Shearing strength
	lb/in.^{2}
Marble	1,300 to 6,500
Slate	2,000 to 3,600
Limestone	800 to 3,600
Sandstone	300 to 3,000

4. ABRASIVE HARDNESS

The abrasive resistance of granite is of interest when the stone is to be exposed to severe conditions of wear, as in pavements. Three methods [2] for determining the "hardness" of stone have been used rather extensively in this country, and the results are expressed so differently that the values obtained by one method cannot be converted, by computation, to those of another. Each method has certain merits in a particular field of testing, but some confusion may arise if the particular test used is not properly designated.

The results given in table 6 were obtained by the method developed at the National Bureau of Standards. This method employs a specimen 2 by 2 by 1 in., and usually three such pieces are tested for each sample. They are held on a revolving cast iron disk under a constant pressure of 2 kg (4.4 lb) and made to revolve at half the speed of the grinding disk. The abrasive is artificial corundum No. 60, which is fed to the disk at a constant rate. Since the abrasive is much harder than quartz, the wear is more rapid than in the Dorry test. The amount of wear is determined by weighing the specimens before and after the test, and the results are computed by the formula

$$H_a = \frac{10(W_s + 2{,}000)\ G}{2{,}000\ W_a},$$

in which W_s is the original weight of a specimen, G its bulk density, and W_a the loss of weight (all weights in grams).

Comparisons of results obtained with the Dorry and National

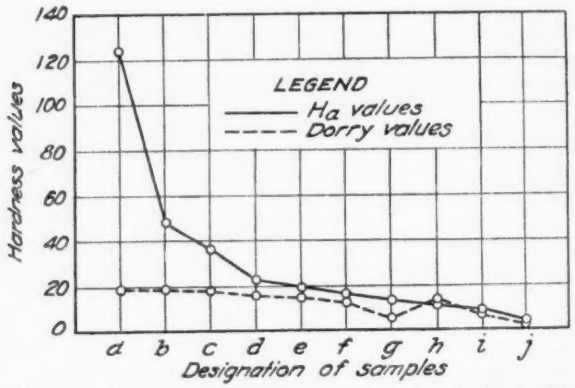

FIGURE 3.—*Abrasive hardness results on 10 materials, obtained with the Kessler and Dorry tests.*

a=serpentine; *b*=granite; *c*, *e*, and *g*=marble; *e*, *f*, *i*, and *j*=limestone; *h*=slate.

Bureau of Standards abrasion machines, on materials ranging in hardness from soft limestones to very hard serpentine, are shown in figure 3. For soft materials the results are of approximately the same magnitude, but for materials of intermediate hardness the H_a values

[2] The Dorry and the Deval tests are described in Office of Public Roads, Bull. 44, U. S. Dept. of Agriculture, Washington, D. C. The third, designed by D. W. Kessler, is described in detail in BS J. Research 11, 637 (1933) RP612, *Wear Resistance of Natural Stone Flooring.*

FIGURE 2.—*Punching shear testing device with specimen in position for test.*

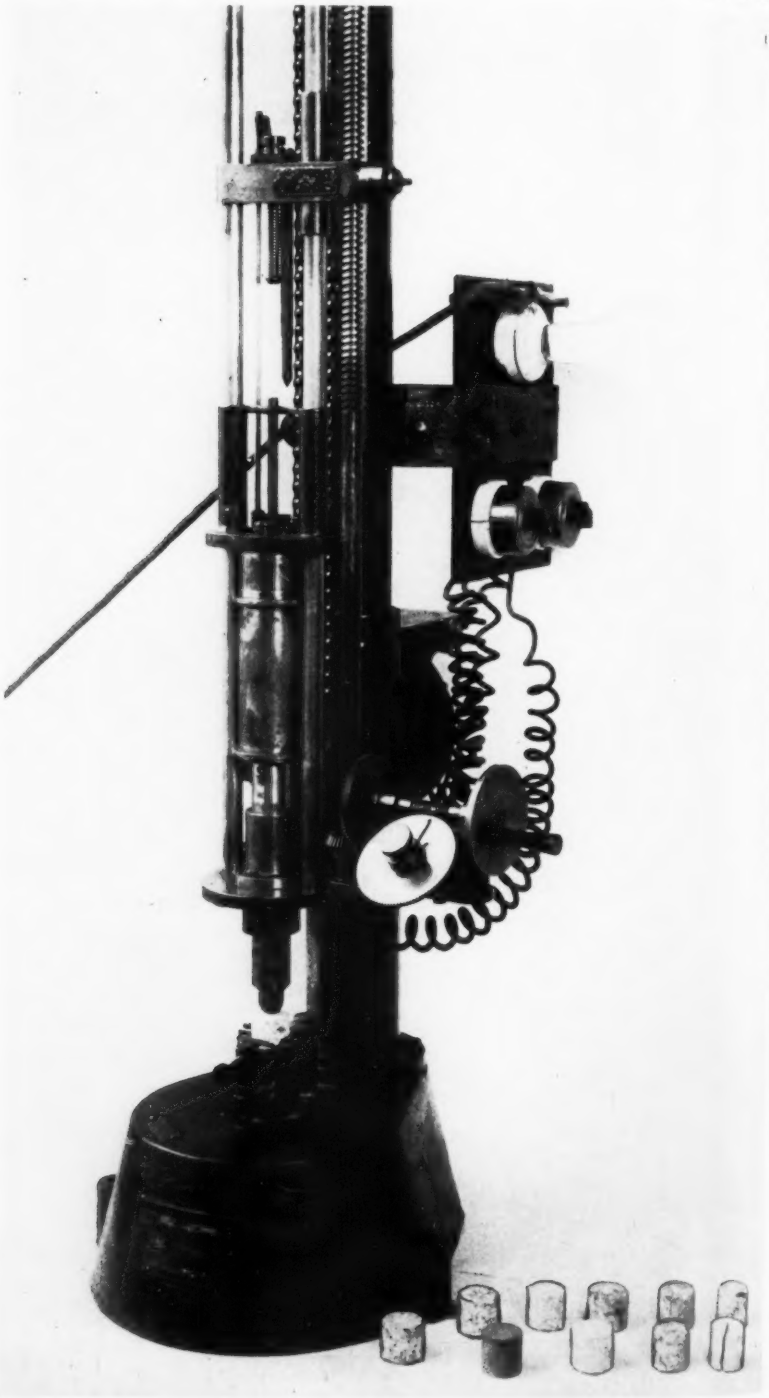

FIGURE 4.—*Page impact toughness tester, showing specimens in position and several specimens after test.*

are about 25 percent higher and large differences are shown on hard materials because the Dorry values cannot be greater than 20.

TABLE 6.—*Results of abrasion tests on granite*

Serial No.	Number of tests	H_a values		
		Maximum	Minimum	Average
11	3	48	41	46
12	3	69	60	63
17	3	72	57	64
18	3	73	60	66
19	3	75	61	66
28	3	55	37	44
29	3	56	50	53
41	3	76	59	68
42	6	98	77	88
53	3	54	45	50
67	11	66	46	54
89	6	96	78	87
109	9	65	43	60
113	6	44	26	37

Comparisons of results (see footnote 2) obtained in the Dorry and Deval tests on 40 samples of granite show that the Dorry values are, on the average, about 20 percent higher than the "French coefficients" of the Deval test. Since the Deval test involves the toughness factor, the ratio between the two varies considerably. Materials that have high toughness values usually show a correspondingly high French coefficient.

The results in table 6 on granites from 14 producing districts show a range in H_a values of 37 to 88, indicating that some granites are much more resistant to abrasion than others. In comparison with most other types of stone these values are all high, and good resistance to wear may be expected from any sound granite in the floors or steps of buildings. Probably the greatest concern regarding granite in such applications should be whether it will wear to a smooth surface and become slippery.

In some of the tests which were repeated on the same specimens, it was noted that the second test gave higher H_a values than the first. This seems to indicate that granite is injured to a slight depth in finishing. Surfacing methods which employ a bush hammer or pneumatic tool are rather severe, and they probably shatter the more brittle minerals to some extent. Specimens of a granite which is used extensively in building construction were tested three times on one face, and the results were as follows:

Test	H_a values
First (on original surface)	41
Second	49
Third	52

The total depth ground off from each specimen was about 0.01 in., and the indicated increase in hardness was nearly 27 percent.

A few hardness tests have been made on the principal minerals of granite which indicate that quartz is more than twice as resistant to wear as the feldspars. The results were as follows:

Material	H_a values
Quartz	180
Orthoclase	53
Albite	81

The following H_a range values for other types of stone are quoted from National Bureau of Standards Research Paper No. 612 (see footnote 2):

Material	H_a values
Marble	8 to 42
Limestone	1 to 24
Sandstone	2 to 26
Slate	6 to 12
Serpentine	13 to 110
Travertine	1 to 16

5. TOUGHNESS

The resistance of a stone to impact is a measure of its tenacity, and this property is commonly called "toughness." Stones of low toughness are apt to spall when sharp corners are exposed to rough usage, such as occurs on steps and on some other members near the base of buildings.

The samples for these tests were selected with the idea of showing the range in toughness values, and the tests were made on a Page impact apparatus (fig. 4). In this apparatus the specimen is mounted securely on a heavy cast-iron base. A steel plunger with the lower end rounded rests on the specimen, and a 2-kg weight is dropped on the plunger by a motor-driven sprocket chain. The height of the first drop is 1 cm and each succeeding drop is increased by 1 cm. Since the force of the impact is concentrated on the center of the test specimen by the hemispherical contact of the plunger, the specimen usually breaks in three or four approximately equal segments.

This test has been used extensively in selecting road, paving, and curb materials. Results of a large number of tests on various types of stone have been published by the United States Bureau of Public Roads.[3]

Table 7 gives data on 10 samples tested perpendicular to the rift, 2 of which were also tested parallel to the rift. Very little difference in toughness is indicated for the two directions. The average values

[3] Albert T. Goldbeck and Frank H. Jackson, Jr., The Physical Testing of Rock for Road Building, U. S. Dept. of Agr., Office of Public Roads, Bul. No. 44 (1912).
Prevost Hubbard and Frank H. Jackson, Jr., Results of Physical Tests of Road-Building Rock, U. S. Dept. of Agr., Bul. No. 370 (1916).
The Results of Physical Tests of Road-Building Rock from 1916 to 1921, Inclusive, U. S. Dept. of Agr., Bul. No. 1132 (1923).

ranged from 8 to 27, the highest being obtained on a comparatively fine-grained granodiorite.

TABLE 7.—*Results of toughness tests on granite*

Serial No.	Direction of impact*	Number of tests	Toughness values		
			Maximum	Minimum	Average
13	*A*	3	11	7	10
21	*A*	3	11	9	9
21	*B*	3	9	8	8
37	*A*	3	13	12	13
55	*A*	3	25	19	22
63	*A*	5	16	11	13
72	*A*	3	8	8	8
89	*A*	3	11	8	9
89	*B*	3	10	8	9
106	*A*	3	10	7	9
109	*A*	3	28	26	27
116	*A*	3	21	16	19

* *A* signifies that the direction of impact was perpendicular to the rift and *B* parallel to the rift.

The following range and average for toughness results on other types of stone have been taken from publications of the United States Bureau of Public Roads (see footnote 3):

Material	Toughness	
	Range	Average
Diorite	6 to 38	23
Basalt	5 to 40	20
Diabase	6 to 50	19
Quartzite	5 to 30	15
Sandstone	2 to 35	10
Limestone	5 to 20	7
Marble	2 to 23	6

6. ELASTICITY

The elasticity of granite in flexure was determined on serial Nos. 12 and 109. The specimens were 4 by 12 by 1 in., and they were supported on knife-edges of the rocker type 10 in. apart, loaded at midspan. The values were calculated by the formula

$$E=\frac{Wl^3}{4\Delta bt^3},$$

in which W is the load in pounds, l is the length of span, Δ is the deflection at midspan for load W, b is the width, and t is the thickness of specimen. Deflection readings were made by means of a deflectometer suspended from four points on the neutral axis of the specimen above the knife-edges.

In these tests 15 specimens of serial No. 12 were used, 5 being prepared in each of 3 directions with respect to the rift, as shown in figure 1. The load-deflection curves are given in figure 5 for one typical test of each group. These curves show that the Hooke's law does not apply for this granite except within a small range of loading. The modulus of elasticity values shown in figure 5 for each test were computed from points on the curve where the stress-strain ratio was nearly constant. The values for serial No. 12 are much lower than any of the published results for elasticity of granite in compression.

One specimen of granite No. 12 was held under a constant load of 100 lb for 10 days. During the first 6 days the deflection increased

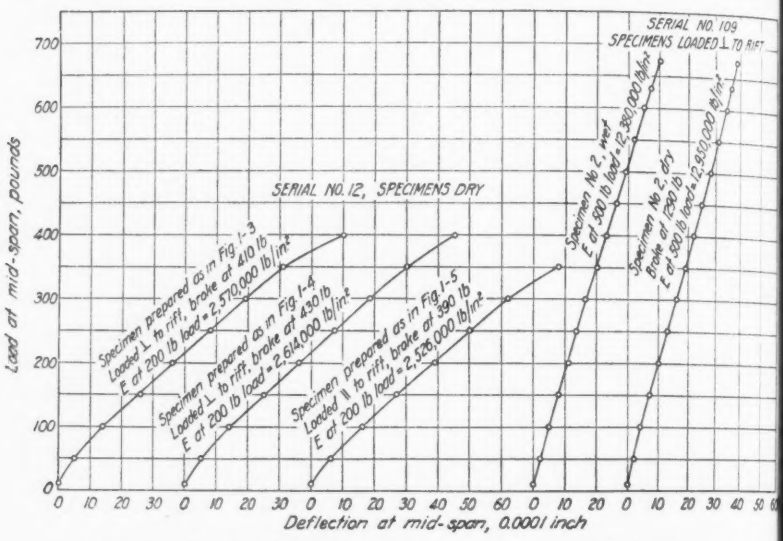
SERIAL NO. 109
SPECIMENS LOADED ⊥ TO RIFT
SERIAL NO. 12, SPECIMENS DRY
Specimen prepared as in Fig. 1-3
Loaded ⊥ to rift, broke at 410 lb
E at 200 lb load = 2,570,000 lb/in.²
Specimen prepared as in Fig. 1-4
Loaded ⊥ to rift, broke at 430 lb
E at 200 lb load = 2,614,000 lb/in.²
Specimen prepared as in Fig. 1-5
Loaded ∥ to rift, broke at 390 lb
E at 200 lb load = 2,526,000 lb/in.²
Specimen No. 2, wet
E at 500 lb load = 12,350,000 lb/in.²
Specimen No. 2, dry
Broke at 1,230 lb
E at 500 lb load = 12,950,000 lb/in.²
Load at mid-span, pounds
Deflection at mid-span, 0.0001 inch

FIGURE 5.—*Load-deflection curves for tests on granite in flexure.*

about 26 percent, after which there was no appreciable change. This seems to confirm the indications from the shape of the curves in figure 5 that such a granite would warp if subjected to flexural stress for a considerable period of time.

Six specimens of serial No. 109 were prepared, three being tested dry and three wet (after soaking 2 days in water). A typical curve for each group is given in figure 5. The results differ widely from those obtained on serial No. 12 in that the proportionality was constant within a considerable range of loading and the modulus of elasticity values were much higher.

Modulus of elasticity determinations made at Watertown Arsenal [4] on five granites show that the ratio of stress to strain in compression increases with increasing loads. The specimens were 4 by 6 by 24 in. and the gage length was 20 in. The stress-strain curves in figure 6 have been reproduced from data obtained in these tests. The results

[4] Report of the Test of Metals and Other Materials for Industrial Purposes at Watertown Arsenal, Mass., Ordnance Dept., U. S. Army, p. 393–401 (1894).

are of considerable interest on account of the unusually large specimens and the large number of strain readings, and therefore some of the data are quoted in table 8.

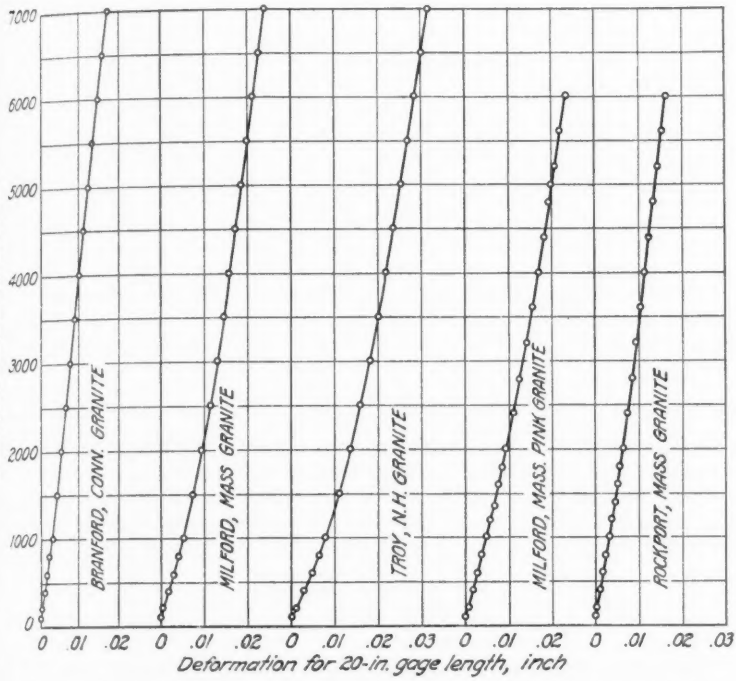

FIGURE 6.—*Load-deformation curves for tests on granite in compression.*

[Reproduced from tests at Watertown Arsenal; Report of the Test of Metals and Other Materials for Industrial Purposes at Watertown Arsenal, Mass., Ordnance Dept., U. S. Army, p. 393–401 (1894).

TABLE 8.—*Modulus of elasticity (E) results on five granites.*

[Tests made at Watertown Arsenal, Report of the Test of Metals and Other Materials for Industrial Purposes at Watertown Arsenal, Mass., Ordnance Dept., U. S. Army, p. 393–401 (1894)]

Source of granite	Load range for which *E* values were computed	Modulus of elasticity in compression (*E*)
	*lb/in.*²	*lb/in.*²
Branford, Conn	1,000 to 3,000	8, 333, 300
Milford, Mass. (pink granite)	1,000 to 2,000	5, 128, 000
Troy, N. H	1,000 to 3,000	4, 545, 400
Milford, Mass	1,000 to 3,000	5, 633, 800
Rockport, Mass	1,000 to 2,000	6, 666, 700

Adams and Coker[5] determined the modulus of elasticity in compression of seven samples of granite, using prisms 1 in. square in cross section and 3 in. long, or, in some cases, cylinders 1 in. in diameter and

[5] Frank D. Adams and Ernest G. Coker, An Investigation Into the Elastic Constants of Rock, More Especially with Reference to Cubic Compressibility, published by the Carnegie Institute of Washington, D. C. (June 1906).

3 in. long. Their tests gave modulus of elasticity values ranging from 5,685,000 to 8,295,000 lb/in.^{2} The series included three domestic granites for which the elasticity values are:

Source of granite	Modulus of elasticity
	lb/in.
Westerly, R. I., granite	7, 394, 000
Quincy, Mass., granite:	
Sample 1	6, 747, 000
Sample 2	8, 248, 000

The stress-strain curves obtained by Adams and Coker are similar to those given in figure 6 in that the stress-strain ratio increases with increasing loads but, on some samples, this tendency was not very pronounced.

7. ABSORPTION

The absorption tests were made on cylindrical specimens 2.1 in. in diameter and 2.5 in. high. Specimens were dried for 24 hr at 105° C and weighed to the nearest 0.01 g before and after immersion periods indicated in table 3; they were surface-dried with a damp towel for the final weighing. The absorption was computed by the formula

$$A=\frac{(W_2-W_1)100}{W_1},$$

in which W_1 and W_2 are, respectively, the original and final weights of the specimens. For most samples the average values stated for the 48-hr and 14-day periods in table 3 represent eight individual tests, whereas the 1-year tests were made on four specimens.

The results indicate that there are small increases in absorption for all samples with the time of immersion up to 1 year. A few samples absorbed approximately 100 percent more water at the end of 1 year than they had absorbed during 48 hr, but for most samples the increases at the end of 1 year were less than 50 percent of the 48-hr values. In general, the normal granites from the midwestern States showed considerably lower absorption than those of the eastern States, and the dark-colored materials (gabbros and basalts) usually showed very low absorption. The average absorption of all samples in these tests for 2 days' immersion was 0.24 percent and for 1 year 0.28 percent. The high values obtained for a few samples (99,103, and 105) which were known to have been taken from near the surface of the ground seemed to indicate that the absorption test may be of value in checking the quality of shipments from any given quarry.

The following ranges in values for certain other types of masonry materials are given for comparison:

Material	Absorption, by weight
	%
Marble [a]	0.06 to 0.45
Slate	0.01 to 0.60
Quartzite	0.1 to 2.0
Sandstone	2.0 to 12.0
Cast stone	2.5 to 18.2
Brick	0.2 to 30.0

[a] This range is based on tests of the usual types of domestic marble. Tests on some of the highly colored recrystallized foreign marbles have given results much higher than 0.45 percent.

8. DENSITY

(a) BULK DENSITY

Bulk-density values (D_b) given in table 3 were obtained in connection with the absorption test. These specimens weighed approximately 400 g, and all weights were determined to the nearest 0.01 g. They were weighed suspended in water after soaking for 48 hr. The results were computed by the formula

$$D_b = \frac{W_1 d}{W_2 - W_3},$$

in which W_1 is the dry weight, W_2 the weight in air after soaking 48 hr, W_3 the weight of soaked specimens suspended in water, and d the density of the water in which the suspended weight was obtained.

The bulk-density values given in table 3 are, for most samples, based on tests of eight specimens. For the entire series of samples, the values range from 2.60 to 3.04, the high value being for a "black granite" (gabbro). Results for the normal granites are mostly between 2.6 and 2.7.

(b) TRUE DENSITY

Density determinations were made on 55-g powdered samples by determining the volumes with Le Chatelier flasks. About 300 g of fragments were selected from specimens after having been crushed in the compression test. These were ground in a mortar until all of the sample passed a No. 100 sieve. The powdered material was dried at 105° C; and after being cooled in a desiccator, it was stored in sealed glass jars until tested. Alcohol was used for the immersion liquid. This method was checked for accuracy against picnometer methods, and it was found that by carefully controlling the temperature of the liquid in the Le Chatelier flasks it gave somewhat closer repeat values on the same sample than the picnometer.

The results are given in table 3. The highest value was 3.080 for a gabbro and the lowest 2.599 for a normal biotite granite.

(c) WEIGHT PER CUBIC FOOT

The weights per cubic foot in table 3 were computed by multiplying the bulk density by 62.35. The following ranges in weight per cubic foot for other types of material are given for comparison:

Material	Weight
	lb/cu ft
Serpentine	158 to 183
Marble	165 to 179
Limestone	117 to 175
Slate	168 to 180
Quartzite	165 to 170
Sandstone	119 to 168

9. POROSITY

Porosity values were computed from the bulk-density and true-density values by the formula

$$P=\frac{(D_a-D_b)\,100}{D_a},$$

in which D_a and D_b are, respectively, the true density and bulk density. This method gives the total pore space regardless of enclosed pores. Microscopic studies indicate that frequently the quartz and sometimes other constituents of granite contain minute cavities which have no outlets. The porosity values obtained on various granites are given in table 3. The average of the series was 1.29 percent and the range 0.40 to 3.84 percent.

The degree of saturation in the various absorption tests may be computed by the formula

$$s=\frac{AD_b}{dP}.$$

For ordinary purposes the value of d may be considered as unity. Computations of the degree of saturation for 45 samples of granite from eastern States for the 1-year absorption tests gave an average of 0.66. Similar computations for 17 samples from midwestern States gave an average of 0.44.

The following range values for porosity of other masonry materials are given for comparison:

Material	Porosity
	%
Marble	0.4 to 2.1
Slate	0.1 to 1.7
Quartzite	1.5 to 2.9
Sandstone	1.9 to 27.3
Limestone	1.1 to 31.0
Cast stone	6.7 to 32.7

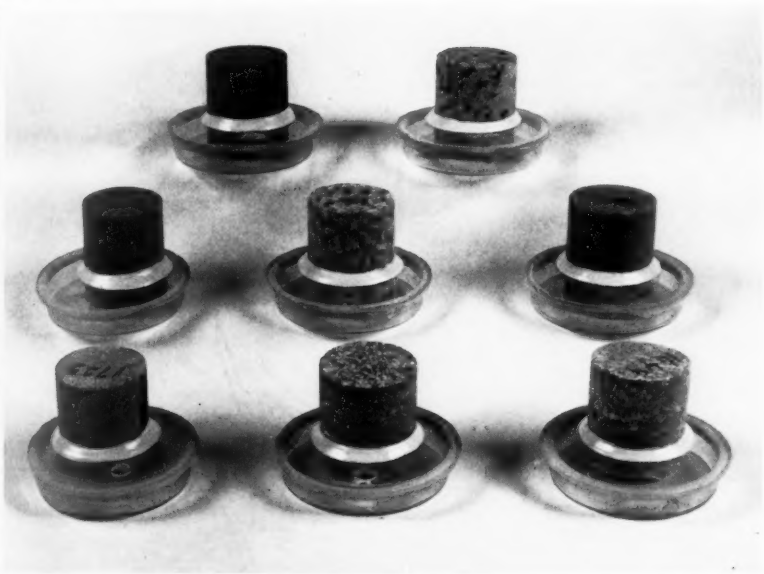

FIGURE 7.— *Water-transmission tests.*

10. WATER TRANSMISSION RATE

Granite is used frequently in the base courses of buildings in which the superstructure is of a more porous material. To a large extent the granite courses prevent the rise of moisture by capillarity from the ground. Differences in porosity or absorption of different granites indicate that some may be better adapted to grade-course construction than others. Hence a test was devised to compare the rates at which different samples allow water to pass through the pores by capillarity.

The tests were made by sealing specimens 2.1 in. in diameter by 2.5 in. high in shallow glass vessels containing a small amount of distilled water (fig. 7), and determining the rates of loss in weight at suitable intervals. In this test no attempt was made to control the tempera-

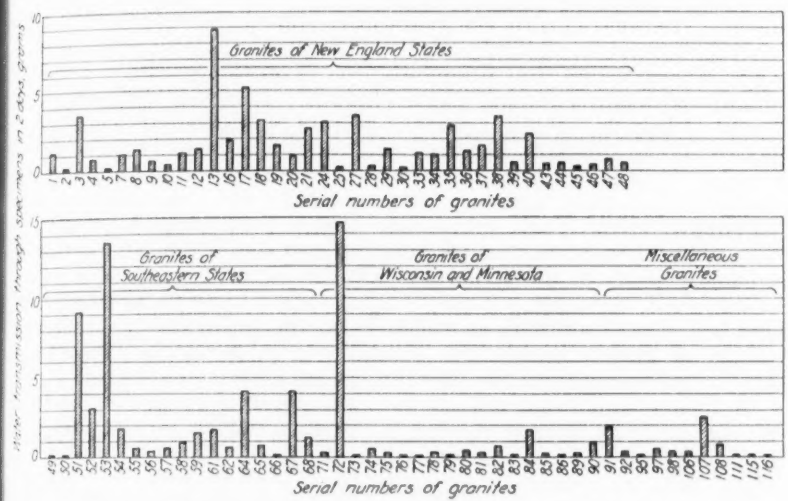
Granites of New England States
Serial numbers of granites
Granites of Southeastern States
Granites of Wisconsin and Minnesota
Miscellaneous Granites
Serial numbers of granites

FIGURE 8.—*Results of water-transmission tests on 85 granites.*

ture or humidity, because moisture passed through the pores so slowly that the surface above the cover plate remained dry. Several tests were made by placing an impervious membrane around the vertical side of the specimen so evaporation could occur only at the top. When this was done, a period of several days was required for the weight losses to reach a constant value and the time was variable for different granites. Without the membrane the rate of loss became constant for most samples in 48 hr, and this period was used for determining the comparative rates. Several samples were weighed daily for 15 days, and a slight decrease in the rate of evaporation occurred after about 3 days.

The results for 85 samples of granite are shown in figure 8. For more than half of the samples the water transmitted in 48 hr was less than 1 g, the greatest amount transmitted was nearly 15 g, and the average of all was 1.62 g. There appears to be no definite relation between the porosity or absorption values and the transmission rates, except that those granites of very low absorption also have low transmission rates. The transmission rates show a much wider range of

values than either the absorption or porosity, and this test may give information on the pore structure that is not indicated by absorption or porosity determinations.

The sample that gave the highest transmission rate (No. 72) was one which showed unmistakable signs of alteration from its original state by weathering. This suggests that the transmission test may be a more definite indicator than the absorption test for determining whether different consignments of granite have been quarried near weathered zones.

V. DURABILITY STUDIES

1. OBSERVATIONS ON STRUCTURES

Inspections of numerous buildings and monuments ranging in age from a few years up to 189 years have shown that granite is a very durable material. A disfiguring, but probably not otherwise serious, type of weathering that frequently occurs on the granite in buildings manifests itself by scaling of the surface. This weathering has been noted mainly on the lower parts of granite buildings, but it sometimes occurs at considerable heights. The scaling is usually more or less patchy, starting with a blister which breaks and then a thin layer scales off. A good example is found on the old Post Office building in Albany, New York, which has blistered and scaled extensively. The New York State Capitol building also shows the same condition on many parts of the structure. So far as can be judged from the cases noted, the scaling is not characteristic of any particular granite and it occurs in southern as well as in northern climates. A memorial tablet of granite inside the Washington Monument, Washington, D. C., has scaled until the inscriptions are almost obliterated.

A theory sometimes offered to explain scaling is that a thin layer of the stone is injured during fabrication, which makes that part less resistant to frost action. Indications of injury to a depth of about 0.01 in. caused by finishing have been referred to in section III–4; but the above theory does not account for scaling on the interior of structures, where neither temperature nor moisture conditions are favorable for frost action. Furthermore, observations indicate that scaling occurs as frequently on modern buildings as on old ones, although modern finishing methods are probably much less severe than those formerly used. The granite for King's Chapel in Boston (built in 1750) was finished by crude and very severe methods, but there is little visible disintegration on this structure.

Blistering and scaling of granite monuments seem to be rare, and only one pronounced example is known to the authors. This is a large monument in Rock Creek Cemetery (Washington, D. C.) which shows scaling over a considerable portion of the surface, and the polished die is so badly scaled that some of the inscriptions are almost illegible.

2. STUDIES OF GRANITE WEATHERING

A few samples of granite from old buildings were studied to determine the effects of weathering. Two blocks from a building 100 years old were cored and tested for absorption and compressive strength. One of the samples was identified as being from the same quarry as serial No. 3 in table 1. The tests showed no loss of strength

or increase in absorption for the weathered sample. The strength and absorption values of the other weathered sample were near the average of these properties obtained on all samples tested, but no newly quarried sample of the same granite was available for a more definite comparison. The results, therefore, indicate that 100 years of weathering had not changed the physical properties of these granites sufficiently to be measurable.

Chemical and microscopic studies have been made on a few samples of weathered granite, and the results showed that variable amounts of gypsum were present. This indicates that granite scaling may be caused by the crystallization of this salt in the pores of surface layers. The amount of such salt in samples taken from exposed parts of structures evidently depends largely on the time of sampling. In summer, a considerable part of the soluble matter may be carried away by rains, hence samples collected in winter are more indicative. Table 9 gives the results of studies on five samples.

TABLE 9.—*Chemical and microscopic studies on weathered granite*

Sample	Source and description of samples	Water-soluble [a]	Sulfur	Calcium sulfate	Calcium carbonate
		%	%	%	%
A	Surface scales from Albany Post Office, collected in winter.	2.2	0.41	1.72	0.67
B	From Albany Post Office, 3 in. back of exposed surface. Collected in winter.	.5	.01	.07	.82
C	Surface scales from Cleveland Post Office where freely exposed to rains. Collected in summer.	1.7			
D	Surface scales from retaining wall of Cleveland Post Office, not exposed to rains.	40.0			
E	Surface scales from Indianapolis Post Office where exposed to rains. Collected in winter.	2.9			

[a] Amounts were determined gravimetrically and the crystals identified with a petrographic microscope as being mainly gypsum.

It will be seen that the surface samples contained variable amounts of calcium sulfate (probably as gypsum), which substance is not found in newly quarried granites. Sample *B* shows that calcium sulfate (probably as gypsum) was present in small amounts at a depth of 3 in. from the surface. Sample *D* from granite masonry not exposed to rains showed a large accumulation of this salt, but the distintegration at that place was no more conspicuous than where sample *C* was taken. The difference in calcium carbonate for samples *A* and *B* may have been due to a natural variation, because sample *A* was taken from various parts of the buildings.

The calcium sulfate [6] in weathered granite may have been formed by the action of sulfurous or sulfuric acids on the calcium carbonate originally present in the granite. Sulfur dioxide from the combustion of coal is usually present in the air, and this gas readily forms acids in the presence of moisture. When a particle of calcium carbonate is converted to gypsum, there is an increase in volume and the latter requires a space approximately twice that of the calcium carbonate. The conversion of calcium-carbonate particles in granite to gypsum would, therefore, cause internal stresses, which may be one reason for granite scaling.

[6] The action of calcium sulfate in the weathering of limestone has been studied by A. Scott Russel, Report of the Building Research Board (1927) Dept. of Sci. and Ind. Res., Westminster, England. The same salt has been found by D. W. Kessler and W. H. Sligh to be a factor in the weathering of slate, *Physical Properties and Weathering Characteristics of Slate*, BS J. Research **9**, 377 (1932) RP477.

The crystal growth of gypsum from sources outside the granite may be a possible cause of granite scaling. About 4 percent by weight of gypsum is used in portland cement, and hence a 1:3 mortar contains approximately 1 percent. A computation will show that 1 volume of such a mortar contains a volume of gypsum almost sufficient to fill the pores of an equal volume of an average granite. If the gypsum is leached from the mortar and concentrated in a surface layer of the granite, it might cause or contribute to scaling. A test, made to determine if granite could be disintegrated by allowing gypsum to crystallize in the pores, gave negative results. In this test several granite specimens were partly immersed in a saturated solution of calcium sulfate for 3 years so that the solution could pass upward through the pores and evaporate on the surfaces exposed to the air. None of the specimens scaled and very little efflorescence occurred.

Experiments made by leaching specimens with a 10-percent solution of sulfuric acid resulted in complete disintegration of one granite within 30 days and a few others within 3 months. Scaling occurred on some of the granites in a manner resembling that found on buildings, but with this relatively strong acid solution the disintegration appeared to be, in part, due to alteration of the biotite. When the disintegrated material was leached with distilled water and the leaching water filtered and evaporated, a considerable amount of gypsum crystals formed in the evaporating dish. This formation of gypsum supports the observation, made in connection with the study of scales from weathered granite, that the conversion of calcite particles in the granite to gypsum is a major cause of scaling. Under actual structural conditions this process may be aided by recrystallization of gypsum from other sources.

3. STUDIES OF WEATHERING AGENTS

(a) FROST ACTION

Freezing tests were made on 14 samples described in table 1 under serial Nos. 6, 14, 26, 53, 54, 60, 67, 69, 70, 87, 103, 104, 105, and 110. The specimens were frozen 4,500 times under conditions simulating those in masonry walls above the ground level. Before the first freezing the specimens were soaked for two weeks in water, then placed in wire baskets (free to drain) in the freezing chamber at a temperature of about −12° C. The freezing period was at least 6 hr. Thawing was done by immersing the specimens in water at about 20° C for 1 hr. After this series of freezings, the test was continued by freezing the specimens 500 times while standing in ½ in. of water.

None of these samples showed any visible signs of disintegration after the total 5,000 cycles, and the water-transmission rates of the specimens were no higher than those of specimens of the same granites not subjected to freezing. The samples in this test were probably no more resistant to freezing than any of the others, but the low strengths and high absorption as well as the mineral alterations on some, especially serial Nos. 103 and 105, indicated that they had undergone considerable weathering before the freezing tests were started. These two granites would be considered of inferior quality as judged by their strength and absorption; but since they, as well as all of the others, showed no disintegration in the prolonged freezing test, it seems evident that frost action is of minor importance in the weathering of granite.

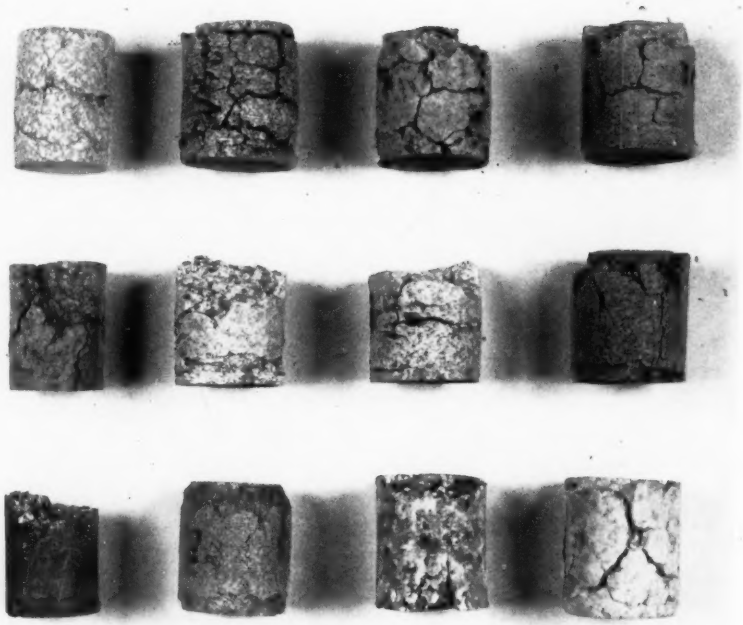

FIGURE 9.—*Specimens after sodium-sulfate crystallization tests.*

(b) CRYSTALLIZATION TEST

The sodium-sulfate crystallization (or soundness) test was applied to 79 samples, and the results are given in table 10. The procedure consisted in soaking cylindrical specimens 2.1 in. in diameter by 2.5 in. high for 17 hr in a saturated solution of sodium sulfate and drying for 7 hr at 105° C. This cycle was repeated until an advanced stage of disintegration occurred in the specimens, as shown in figure 9. A similar test is used frequently in selecting aggregate materials for concrete, and the usual requirement is that the samples shall pass five cycles without losing more than 10 percent of the original weight. None of the samples of granite failed in less than 14 cycles and the average was 42.

This test may give an indication of the relative resistance of different granites to weathering caused by the crystallization of water-soluble salts in the pores as pointed out in section IV–2. The results obtained agree with those of other investigators in that no correlation with freezing tests was found.

TABLE 10.—*Results of crystallization test on various granites (sodium sulfate), by States*

Serial No.	Number of cycles	Nature of disintegration
		MAINE
1	39	Deep cracks and considerable crumbling.
2	70	Crumbled at ends and scaled on sides.
3	16	Deep cracks and considerable crumbling.
4	20	Do.
5	57	Crumbled at ends, discolored (brown).
7	21	Deep cracks.
8	27	Deep cracks and considerable crumbling.
9	19	Deep cracks.
10	45	Deep cracks at ends.
11	39	Deep cracks one end.
12	17	Deep cracks and considerable crumbling.
13	23	Ends crumbled.
		NEW HAMPSHIRE
18	20	Deep cracks and considerable crumbling.
19	20	Do.
20	39	Do.
21	20	Deep cracks.
24	23	Crumbled at ends.
		VERMONT
25	39	Deep cracks, much crumbling.
27	16	Deep cracks.
28	20	Do.
29	14	Deep cracks, much crumbling.
30	179	No decay, some discoloration.
		MASSACHUSETTS
33	16	Deep cracks.
34	16	Do.
35	21	Do.
36	20	Deep cracks, considerable crumbling.
37	24	Deep cracks.
38	25	Do.
39	54	Deep cracks, much crumbling.
40	19	Deep cracks.

TABLE 10.—*Results of crystallization test on various granites* (*sodium sulfate*), *by States*—Continued

Serial No.	Number of cycles	Nature of disintegration
		CONNECTICUT
43	27	Deep cracks along rift.
44	25	Deep cracks.
45	45	Surface scaled to slight depth.
46	29	Deep cracks, much crumbling.
47	26	Deep cracks, ends crumbled.
48	18	Deep cracks, considerable crumbling.
		NORTH CAROLINA
49	179	No appreciable decay.
50	179	One end cracked, slight decay.
51	17	Deep cracks.
52	14	Deep cracks, considerable crumbling.
53	19	Deep cracks.
54	16	Do.
55	97	Deep cracks, considerable crumbling.
56	155	Do.
57	26	Do.
58	33	Do.
		SOUTH CAROLINA
59	19	Deep cracks.
61	25	Do.
62	56	Completely disintegrated.
64	18	Deep cracks, considerable crumbling.
		GEORGIA
65	39	Deep cracks, considerable crumbling.
66	29	Deep cracks at ends.
67	19	Deep cracks.
68	24	Do.
		WISCONSIN
71	296	One end crumbled.
72	17	Deep cracks.
73	77	One end crumbled, several cracks.
74	39	Deep cracks, much crumbling.
75	23	Some cracks, ends crumbled.
76	66	Ends crumbled.
77	27	Many deep cracks.
		MINNESOTA
78	25	Many deep cracks.
79	66	Ends crumbled.
80	24	Many deep cracks.
81	25	Do.
82	79	Ends crumbled, some cracks.
83	39	Deep cracks, much crumbling.
84	27	Many deep cracks.
85	20	Deep cracks, considerable crumbling.
86	18	Do.
90	39	Ends crumbled.
		MISCELLANEOUS
91	18	Deep cracks, considerable crumbling.
92	18	Do.
95	155	Scaled off to slight depth.
97	56	Ends crumbled, surface pitted.
98	32	Cracked along rift.
106	17	Deep cracks.
107	20	Do.
108	17	Deep cracks, considerable crumbling.

(c) TEMPERATURE AND MOISTURE CYCLE TESTS

Forty-seven samples were subjected to a test in which the specimens were soaked in water for 17 hr, then dried at 105° C for 7 hr. This cycle was repeated 1,000 times, the object being to determine if any mineral alterations occur within the stone which bring about disintegration. The changes were studied by means of weight, absorption, and water-transmission tests. The results are given in table 11. Most samples showed slight increases in weight, considerable increases in absorption, and large increases in water-transmission rates. The original water-transmission rate was not determined on the same specimen that was subjected to soaking and drying, but on another core from the same block of stone.

TABLE 11.—*Changes in absorption, weight and water transmission of various granites, by States, after 1,000 cycles of soaking and drying*

Serial No.	Absorption			Change in weight	Water transmission		
	Original	Final	Change		Original	Final	Change
MAINE							
	%	%	%	%	g	g	%
1	0.23	0.29	+26	+0.05	1.19	1.72	+45
3	.26	.30	+15	+.03	3.57	26.93	+650
7	.21	.23	+10	+.04	1.04	5.34	+410
8	.23	.26	+13	+.04	1.28	11.39	+790
9	.23	.26	+13	+.02	0.57	17.79	+3000
10	.22	.26	+18	+.06	.38	1.89	+400
11	.31	.37	+19	+.04	1.15	21.29	+1750
13	.32	.37	+16	+.06	8.84	21.58	+144
16	.32	.42	+31	+.04	1.89	22.89	+1100
NEW HAMPSHIRE							
17	0.29	0.38	+31	+0.08	5.24	16.21	+210
18	.23	.28	+22	−.04	3.17	16.85	+430
20	.34	.65	+91	+.06	0.89	23.20	+2500
21	.29	.45	+55	+.06	2.64	34.69	+1200
VERMONT							
25	0.20	0.26	+30	+0.05	0.21	1.10	+424
27	.26	.37	+42	+.02	3.45	14.48	+320
28	.27	.30	+11	+.01	0.29	.80	+176
MASSACHUSETTS							
33	0.26	0.32	+19	+0.03	1.15	15.17	+1200
34	.25	.36	+24	+.01	0.98	14.05	+1330
35	.21	.28	+33	.00			
38	.32	.42	+31	+.06	3.43	4.28	+25
40	.22	.33	+50	+.04	2.33	12.69	+440
CONNECTICUT							
43	0.19	0.41	+115	+0.04	0.48	2.83	+490
45	.22	.29	+32	+.05	.30	7.27	+2300
46	.31	.45	+45	+.06	.38	16.85	+4300
NORTH CAROLINA							
51	0.20	0.29	+45	+0.01	9.16	28.92	+220
53	.25	.29	+16	+.03	13.49	18.59	+38
54	.29	.31	+7	+.04	1.79	12.51	+600

TABLE 11.—*Changes in absorption, weight and water transmission of various granites, by States, after 1,000 cycles of soaking and drying*—Continued

Serial No.	Absorption			Change in weight	Water transmission		
	Original	Final	Change		Original	Final	Change
				SOUTH CAROLINA			
	%	%	%	%	g	g	%
59	0.25	0.39	+56	+0.05	1.55	17.96	+1060
61	.26	.34	+31	+.06	1.75	22.09	+1160
				GEORGIA			
65	0.30	0.43	+43	+0.08	0.72	12.54	+1640
66	.24	.39	+63	+.07	.14	21.02	+14900
67	.19	.28	+47	+.03	4.17	27.40	+560
68	.26	.35	+35	+.06	1.21	21.85	+1700
				WISCONSIN			
71	0.05	0.10	+100	−0.01	0.24	1.83	+660
72	.30	.28	−7	+.01	14.69	23.20	+58
74	.12	.19	+58	.00	.49		
76	.05	.09	+80	.00	.09	0.25	+180
77	.08	.16	+100	.00	.10	2.57	+2470
				MINNESOTA			
79	0.11	0.17	+55	−0.01	0.16	5.49	+3300
80	.10	.14	+40	−.02	.42	.48	+14
81	.10	.15	+50	−.04	.29	2.00	+590
82	.19	.26	+37	+.05	.70	2.07	+196
83	.07	.11	+57	−.02	.14	1.18	+740
84	.19	.27	+42	+.05	1.68	5.26	+210
				MISCELLANEOUS			
91	0.22	0.27	+23	+0.02	1.89	16.58	+780
106	.14	.18	+29	−.07	0.36	3.01	+740
107	.15	.21	+40	−.03	2.51	8.79	+250

In some specimens cracks were plainly visible. Several specimens did not show cracks; but when water passed through them, it caused several narrow bands of discoloration on the surface, indicating that small cracks had developed. Such results, along with the large increases in transmission rates for most samples, seem to show that the granites undergo changes caused by temperature and moisture cycles when the temperatures are above the freezing point. However, none of the specimens showed the scaling condition referred to as occurring on the lower courses of granite buildings. Most of the specimens increased in weight which indicates that the changes may have been caused by hydration, oxidation, or sulfation. Apparently there is no relation between the results of this test and the amount of calcite or pyrite present in the sample.

In order to determine if a temperature cycle alone would show effects like those observed in the soaking and drying test, 3 specimens were subjected to 2,000 heating and cooling cycles. The specimens were of the same type and size as those used in the soaking and drying

tests. They were heated to 105° C and cooled to approximately −10° C. Except for a small amount of condensation on the surface when the specimens were removed from the freezing chamber, there was no moisture involved in these tests. None of the specimens showed cracks or increases in water-transmission values, and apparently there was no injury to the granite.

VI. SUMMARY

Samples of granite were collected from 17 States for a comparative study of such characteristics as are of interest in connection with various uses of this material. Microscopic studies were made on samples of those granites for which data on the mineral constituents were not available in the literature. Tests for compressive strength and absorption were made on 116 samples and their porosity and density determined. Strength was determined on specimens in the dry and in the wet condition and also by applying the load perpendicular and parallel to the rift. Absorption tests were made for three periods of immersion, namely 2 days, 14 days, and 1 year. The flexural strength, shearing strength, modulus of elasticity, toughness, and abrasive hardness, were determined on a few selected samples. Elasticity in flexure of two samples was studied under various conditions of loading, and typical load-deflection curves for these are given. Durability studies were made by means of various laboratory tests as well as by observations on structures.

The compressive strength for individual specimens ranged from 5,900 to 60,000 lb/in.^{2} and averaged 24,500 lb/in.^{2} for specimens tested dry and loaded perpendicular to the rift. The results show that, for some granites, different test pieces of the same sample vary considerably in strength, but the difference between the average of all tests on a sample and the highest or lowest individual result was usually less than 10 percent. Although some granites appear to be stronger when loaded perpendicular to than when loaded parallel to the rift, these differences were not sufficiently consistent to be significant. Compressive strengths of wet specimens averaged about 12 percent lower than those of dry specimens.

The porosity values ranged from 0.40 to 3.84 percent and averaged 1.29 percent. Absorption tests show that the specimens did not become saturated with water even after 1 year of immersion, and for a larger portion of the specimens the pores were less than half filled at the end of this time.

A test was devised for comparing the rates at which moisture rises through the pores of various granites by capillarity. The results are believed to give information on pore structure that is not indicated by absorption or porosity tests. This test was used as a measure of deterioration in some of the weathering tests, and it may be useful as a means of checking the uniformity of granite shipments.

Extensive freezing tests have indicated that granite is not seriously affected by frost action. Samples of weathered granite from three buildings were studied to determine the causes of scaling. These studies indicated that the formation of calcium sulfate, from the reaction of acids of sulfur in the air on calcareous minerals in the granite, is the cause of scaling.

Important structures, showing the use of various granites, have been listed as an aid in future studies of the performance of these materials in service.

The authors express their appreciation to the granite producers for cooperation in supplying samples, to J. J. Tregoning for chemical studies and assistance in the petrographic work, and to the United States Bureau of Public Roads for abrasion tests on certain materials with the Dorry apparatus.

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EFFECT OF SPEED OF PULLING JAWS ON THE TENSILE STRENGTH AND STRETCH OF LEATHER

By Robert B. Hobbs

ABSTRACT

The purpose of this investigation was to obtain data on the effect of the speed of the pulling jaws of the testing machine on the tensile strength and stretch of leather. In practice, speeds as low as 0.25 in./min and as high as 12 in./min have been used in different laboratories. In this study, 1,900 measurements of strength and stretch were made at 6 different jaw speeds, using a horizontal Scott machine of 800-lb capacity. The tensile strength increased with jaw speed, the rate of increase being least between 6 and 12 in./min. The change in stretch with jaw speed was erratic; the average difference between the stretch at 0.5 in./min and that at 20 in./min was 1 percent, or no greater than the smallest interval of stretch measured.

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I. INTRODUCTION

Among the physical properties of leather which are most frequently measured are its tensile strength and stretch. The importance of their measurement is indicated by the fact that 10 of the 12 Federal specifications for different types of leather include requirements for these properties. The 10 specifications are those for bag, belting, case, chamois, harness, hydraulic packing, lace, rigging, strap, and upholstery leathers. The specifications for sole and welting leathers alone contain no requirements for strength and stretch.

In view of the widespread use of these measurements, it is desirable that the methods used in different laboratories should give concordant results and that the results in any laboratory should not vary excessively from time to time. Some factors may affect the results seriously. Among them are the method of selecting samples for the test, the temperature and humidity of the atmosphere to which the samples are exposed before and during the test, and the kind of testing machine and manner of using it. This study deals with one phase of the last factor, namely the effect of the speed of the jaws of one type of testing machine.

In different laboratories the jaw speeds that are used vary greatly. In some, the speed of the pulling jaws is as low as 0.25 in./min; in others, as high as 12 in./min. When the machine is operated by hand, the speed of the jaws may change from time to time.

This work was undertaken for the purpose of obtaining data that would show the magnitude of the effect of the speed of the pulling jaws on tensile strength.

II. DESCRIPTION OF LEATHERS USED

The heavy leather used for the main part of this work was made in the experimental tannery at the National Bureau of Standards. The equipment used has been described by Bowker in a recent article.[1] Six steer hides which had been limed and unhaired were obtained from a commercial tannery. They were washed in clean water, bated, and then tanned with a blend of approximately one-third chestnut, one-third cutch, and one-third sulfited quebracho. The pH of the tanning liquors was originally about 5.4, and was reduced at the rate of about 0.05 pH unit per day until it was about 4.0, using acetic acid when necessary. After tanning, the leather was washed thoroughly with fresh water, swabbed with sperm oil, allowed to dry, then dampened again and rolled. No filler or finishing material was used. The completed leather had a pH of about 3.9 and a shrinkage temperature of about 78° C.

Smaller quantities of two commercial leathers were available. One was a vegetable-tanned sole leather, the other a light, snuffed-grain, chrome-tanned leather.

III. PLAN OF THE EXPERIMENTS

In the main part of this study, it was planned to make measurements of tensile strength and stretch at six different jaw speeds, 0.5, 1, 3, 6, 12, and 20 in./min. The method of dividing the leather into the six groups which were necessary was devised with the purpose of making the six groups as nearly uniform as possible. Two sources of variability in the leather were considered, namely the differences between hides and the differences caused by location in the hide.

From each hide were cut 12 blocks for tanning, each block about 14 in. wide by about 16 in. long. After tanning, each block was cut in half along a line parallel to the backbone. This gave 24 pieces from each hide. From each piece were cut 12 tensile-strength test specimens, the lengths of which were perpendicular to the backbone. Thus, there were 288 test specimens from each hide, or a total of 1,728 from the six hides.

Figure 1 represents the location of the test specimens in a hide. The numbers which were given to the 24 pieces for identification are placed on the corresponding pieces. The division of each piece into 12 specimens for tensile-strength measurements is also indicated. The meaning of the shading on some of the strips will be explained later.

The 12 test specimens from each piece were assigned to the 6 groups in pairs, that is, the first and twelfth specimens to one group, the second and eleventh to the next group, and so on. Table 1 shows this assignment for the two pieces, one on either side of the backbone, nearest the tail of hide 1. The speed numbers in table 1 are merely identifying numbers of the groups of specimens, not the actual speeds

[1] Hide and Leather **97**, No. 23, 11 (1939).

used. Since the tensile strength within a piece tends to change regularly from one end of the piece to the other, this assignment of specimens minimizes the variation caused by differences within the piece as a factor affecting the average tensile strength of a group of specimens.

It is still necessary, however, to make allowance for the variation caused by differences in location in the hide. To do this, data obtained from measurements of tensile strength, not yet published, were

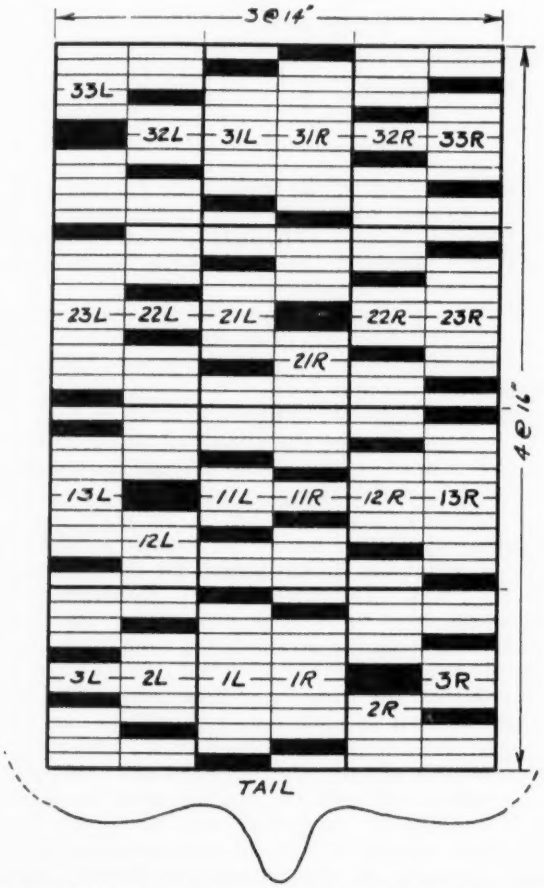

FIGURE 1.—*Method of dividing a hide, with assignment of test specimens to one speed.*

used as a basis for dividing the 24 pieces into 6 groups of 4 pieces each, such that the average tensile strength of each of the 6 groups would be approximately equal to the average strength for the whole hide. The groups of pieces were as follows:

1. 33L, 21R, 12L, 2R.
2. 32R, 22L, 11R, 3L.
3. 32L, 22R, 11L, 3R.
4. 33R, 21L, 12R, 2L.
5. 31L, 23R, 13L, 1R.
6. 31R, 23L, 13R, 1L.

TABLE 1.—*Method of assigning specimens from a block to groups for different speeds*

1L		*1R*	
Strip No.	Speed No.	Strip No.	Speed No.
12	1	12	6
11	2	11	1
10	3	10	2
9	4	9	3
8	5	8	4
7	6	7	5
6	6	6	5
5	5	5	4
4	4	4	3
3	3	3	2
2	2	2	1
1	1	1	6

The two pieces represented in table 1 are *1L* and *1R* from hide 1. The distribution of test specimens from the other pieces of hide 1 in the same group as *1L*, namely *31R*, *23L*, and *13R*, was the same as for *1L*. Similarly, the pieces in the group with *1R* were distributed in the same way as *1R*.

The shaded strips in figure 1 represent the specimens from hide 1 which were assigned to the group of specimens broken at the first speed. The other specimens were assigned to the groups broken at the other speeds in cyclic order, as may be seen by comparing the two pieces in table 1 with pieces *1L* and *1R*, in figure 1, as an example.

The identifying numbers of the speeds of the groups to which the specimens in hide 2 were assigned, were obtained from the numbers of the speeds for the similarly located specimens in hide 1 by performing the cyclic substitution (123456)—that is, for each speed number on hide 1 was substituted the number immediately following it in the symbol (123456) to get the speed number for the corresponding specimen on hide 2 (1 was substituted for 6). Repeating this operation gave the speed numbers for hide 3, and so on. By this method of sampling, the average tensile strength for each speed has been freed as far as possible from the effects of variations between the hides.

It was desired that the number of specimens of each of the commercial leathers broken at each speed should be, as nearly as convenient, equal to the number from each of the experimental hides at each speed. For this reason, the material available from the commercial leathers was sufficient for making tests at two speeds only. From the sole leather 68 specimens were obtained; from the light leather, 108. Test specimens were cut perpendicular to the backbone, with alternate specimens assigned to each of the two speeds selected.

The necessity of measuring the strength and stretch of leather under conditions of controlled humidity was shown by Veitch, Frey, and Leinbach.[2] Accordingly, all work was done in a room where the temperature was controlled at approximately 72° F and the relative humidity at about 65 percent. The test specimens were cut out with a metal die in the standard dumbbell shape [3] (fig. 2). The leather was removed from the conditioning room for a period of time just long enough to cut out the specimens, which were then quickly returned to

[2] J. Am. Leather Chem. Assn. **17**, 492 (1922).
[3] J. Am. Leather Chem. Assn. **32**, 418 (1937).

the conditioning room. The test pieces were then placed in racks allowing free access of air to both sides, where they were allowed to stand for 3 days before the thickness was measured and for at least 3 more days before they were broken.

The thickness of the leather was measured with a Randall & Stickney gage graduated in 0.001 in. and having a contact foot 1 cm in diameter. The load on the foot was approximately 1 lb. The minimum thickness of the reduced section of the specimen was used in calculating the tensile strength. The specimens varied in thickness from about 0.10 in. to about 0.25 in.; the thickness was read to 0.001 in.

Two transverse gage marks were placed on the reduced section of the specimen equidistant from its center, and 2 in. apart. The distance between the gage marks was measured with calipers as the specimen

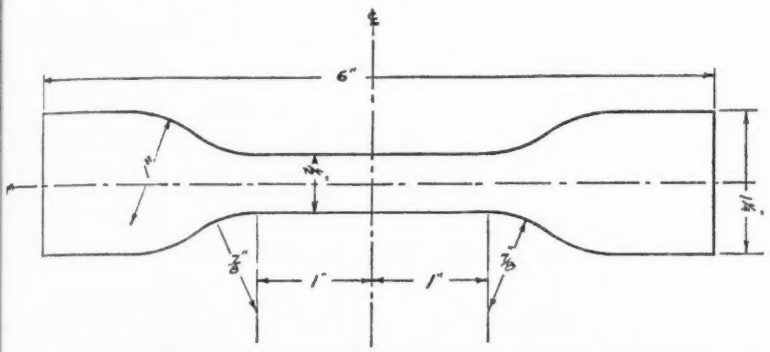

FIGURE 2.—*Size and shape of specimen for test.*

was being stretched; the distance when the specimen broke was recorded to the nearest 0.02 in. This figure was used in computing the percentage elongation.

The machine used for breaking the specimens, which was set up in the conditioning room, is a horizontal Scott machine of the pendulum type, having a capacity of 800 lb. A calibration of this machine showed that the errors of its indicated loads did not exceed 1.4 percent for the range 200 to 800 lb. The pulling jaws are driven by an electric motor, through a belt and pulleys, the size of which could be varied to change the speed of the jaws. The speed of the pulling jaws, which was observed when they were running free, was computed from the time required for a given point on the jaws to travel a measured distance. It is estimated that the measurements of speed were accurate within 1 percent. The speeds chosen for study were 0.5, 1, 3, 6, 12, and 20 in./min, which roughly cover the range of speeds which have been used in practice.

The observed tensile strength depends on the rate of loading of the specimen, and this, in turn, depends not only on the speed of the pulling jaws but also on the capacity of the machine. Therefore, control of the speed of the pulling jaws is not in itself sufficient to ensure duplication of results on different machines. It is also necessary that the capacities of the machines should be the same. When considering the results, then, it should be remembered that they are applicable only to a machine of the type and capacity used in this work.

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IV. RESULTS

1. TENSILE STRENGTH OF EXPERIMENTAL LEATHER

The results of the measurements of tensile strength on the leather prepared in the experimental tannery are given in table 2. Each figure in the body of the table is the mean of all measurements on specimens from the indicated hide at the given speed. At the foot of each column is given the mean of all measurements at the given speed. Of the 1,728 specimens, 57 were made worthless by cuts, grub holes, and other flaws in the leather, leaving 1,671 valid observations. Thus, each of the 6 means represents about 278 observations; the minimum was 272, the maximum, 281.

TABLE 2.—*Tensile strength of experimental leather tested at different jaw speeds*

Hide number	Speed of pulling jaws (in./min)					
	0.5	1	3	6	12	20
	lb/in.²	*lb/in.²*	*lb/in.²*	*lb/in.²*	*lb/in.²*	*lb/in.²*
1	4,377	4,440	4,481	4,583	4,699	4,735
2	4,371	4,271	4,374	4,515	4,657	4,857
3	3,987	3,954	3,948	4,080	4,026	4,155
4	4,730	4,855	4,857	4,958	4,926	5,233
5	4,311	4,433	4,681	4,613	4,560	4,774
6	4,386	4,435	4,615	4,677	4,708	4,664
Mean of individual observations	4,368	4,404	4,490	4,573	4,597	4,733

The mean tensile strengths are plotted against the corresponding jaw speeds in figure 3. The rate of change of tensile strength is greatest between 0.5 and 1 in./min, and decreases to a minimum between 6 and 12 in./min. Between 12 and 20 in./min it increases.

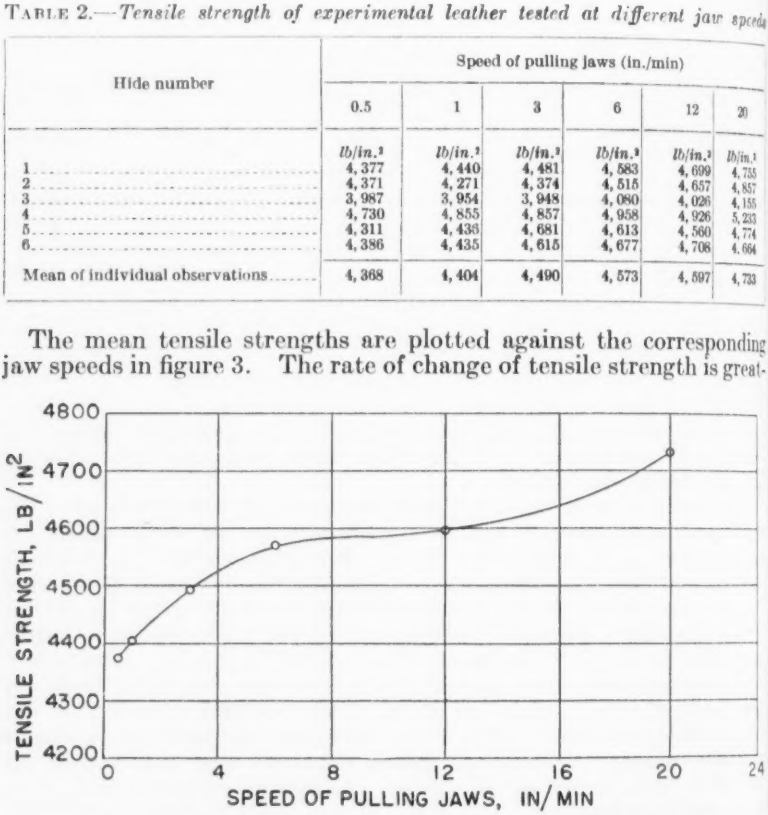

FIGURE 3.—*Effect of speed of pulling jaws on tensile strength.*

The question arises whether the observed differences in tensile strength at the different speeds are statistically significant. A test of significance which is frequently used is the comparison of the quantity tested (in this case, the difference in tensile strength), with its standard deviation. The standard deviation of the difference between the

mean tensile strengths for any two given speeds may be calculated from the original data as follows. For each block of leather there were two measurements of tensile strength at each speed. The average of each pair of measurements for each speed is taken. The difference between two averages from the same block for any two speeds is an estimate of the difference in tensile strength caused by the difference in speed. Since both of the averages from which the difference was derived represent the same block of leather, the difference is an estimate in which the effect of variation between the hides does not appear and the effect of location on tensile strength has been minimized. One such estimate has little value, but a large number of them may be combined to give a reliable estimate. For these data, there are 144 such differences for each pair of speeds, minus those which must be omitted because of missing observations. From each group of differences the mean difference and the standard deviation of the mean difference were then calculated. The ratio of the mean difference to its standard deviation was then computed. These data are given in table 3. The last column of this table gives the probability, in percent, of the chance occurrence of a difference as large as, or larger than, the given difference; this probability is determined by the ratio in the fifth column. (Tables of these probabilities are given in textbooks on statistics and in chemical handbooks.)

The difference between the mean tensile strength at 12 in./min and that at 20 in./min is undoubtedly significant. The difference between 6 and 12 in./min is probably not significant. For other comparisons, the probability of the differences not being caused by chance is great enough to justify the assumption that the effect of jaw speed may be important. Therefore, measurements of tensile strength should be made at a specified speed. Since the rate of change of tensile strength is least between 6 and 12 in./min, the effect of accidental variations in speed would be minimized by selection of a speed within this range.

TABLE 3.—*Test of significance of differences between tensile strengths at different jaw speeds*

Speeds compared	Number of differences	Mean difference	Standard deviation of mean difference	Ratio of mean difference to standard deviation	Probability
(in./min)					*(Percent)*
0.5 and 1	127	52.3	42.2	1.24	21.5
1 and 3	129	108.3	42.6	2.54	1.1
3 and 6	134	71.6	42.6	1.68	9.3
6 and 12	133	24.2	37.7	0.64	52.2
12 and 20	133	170.0	40.4	4.21	0.0

2. STRETCH OF EXPERIMENTAL LEATHER

The results of the measurements of stretch made on the leather tanned at the Bureau are given in table 4, which is arranged similarly to table 2. From the values for the average stretch at each speed, there appears to be a tendency for the stretch to increase as the jaw speed decreases. However, when the data in table 4 for the different hides are compared, it is seen that they are very erratic. Hides 3 and 6 show the stretch tending to decrease as the jaw speed decreases, and

hides 1 and 2 show an increase in stretch only at the two lowest speeds. Moreover, the smallest interval that was read in measuring the stretch was 1 percent, and the six averages lie within a range of 1 percent. For this reason, together with the lack of stability of the data, it did not appear necessary to make a statistical analysis to show that there was no significant effect of the speed of the pulling jaws on the stretch.

TABLE 4.—*Stretch of leather at different jaw speeds*

Hide number	Speed of pulling jaws (in./min)					
	0.5	1	3	6	12	20
	%	%	%	%	%	%
1	32.1	30.3	28.8	29.8	28.7	29.6
2	34.1	31.8	30.7	31.1	30.9	31.6
3	29.5	28.9	28.7	30.1	29.8	29.0
4	29.5	33.0	29.2	30.0	28.6	27.9
5	30.1	30.4	30.4	30.6	30.1	29.0
6	29.0	29.4	30.4	29.5	30.8	31.1
Mean	30.7	30.6	29.7	30.2	29.8	29.7

3. LIGHT LEATHER

One hundred and eight specimens of a light, chrome-tanned leather were broken, 54 at each of 2 speeds, 12 and 20 in./min. The average tensile strength at the lower speed was 4,620 lb/in.^{2}; at the higher speed, 4,800 lb/in.^{2} The difference, expressed as percentage of the strength at 12 in./min, was 3.9 percent. This is comparable to a similarly obtained figure, for the experimental leather, of 3.0 percent.

The average stretch was 38.9 percent at 12 in./min and 36.8 percent at 20 in./min. As in the case of the experimental leather, the stretch is higher at the lower speed, but the difference is not large when compared with the smallest interval of stretch measured.

4. COMMERCIAL SOLE LEATHER

Sixty-eight specimens of commercial vegetable-tanned sole leather were broken, 34 at each of 2 speeds, 0.5 and 12 in./min. At 12 in./min the average tensile strength was 4,290 lb/in.^{2} and the stretch 20.2 percent. At 0.5 in./min the tensile strength was 3,890 lb/in.^{2} and the stretch 22.7 percent. With respect to each property the change was in the same direction as it was for the experimental leather, but the effect seems to have been greater for the sole leather.

V. CONCLUSION

The large differences found between the tensile strengths of leather at different jaw speeds, the capacity of the testing machine being constant, indicate that better duplication of results will be obtained if all measurements are made at the same speed. For the machine used, having a capacity of 800 lb, the preferable speed would seem to lie in the neighborhood of 6 to 12 in./min, since the rate of change of tensile strength is least in this range, and accidental variations in the speed of the jaws would be likely to introduce the least error into the determination if the speed is within this range.

WASHINGTON, June 11, 1940.

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EFFECT OF RATE OF HEATING THROUGH THE TRANSFORMATION RANGE ON AUSTENITIC GRAIN SIZE

By Samuel J. Rosenberg and Thomas G. Digges

Data are presented to show how the rate of heating through the transformation range affects the austenitic grain size at various temperatures of high-purity alloys of iron and carbon and commercial and experimental plain carbon steels. The commercial steels included heats produced under furnace practice that resulted in both controlled and noncontrolled grain size. In certain cases, the austenitic grain size of these alloys and steels was affected by the rate of heating through the transformation range, but all the materials did not respond in the same manner.

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I. INTRODUCTION

The influence of austenitic grain size on many properties of steel has been widely discussed during the past decade and is now generally recognized. Not so much attention has been paid, however, to the study of factors which influence the austenitic grain size. Probably this has been due, at least in part, to the belief that a steel, once made, had what was termed an "inherent" grain size. The word "inherent" denotes unchanging, and its unfortunate use conveyed the idea to many that at any selected temperature a particular steel had a definite grain size and that this remained fixed regardless of other factors.

In a previous study of the influence of austenitic grain size on the hardenability of high-purity iron-carbon alloys [1],[1] it was observed that the rate of heating through the transformation range often had a pronounced influence on the final austenitic grain size. If steels are similarly affected, this factor deserves consideration. This becomes especially important when consideration is given to the wide range

[1] Figures in brackets indicate the literature references at the end of this paper.

in heating rates commercially employed, varying from the extremely rapid heating secured in induction hardening to the slow rates obtained with furnace heating of large parts.

The present study was made to show the effect of the rate of heating to different temperatures on the austenitic grain size of high-purity alloys of iron and carbon and of commercial plain carbon steels, with a wide range in carbon content. The commercial steels also differed in the method of deoxidation. The effects of other factors that influence the austenitic grain size, such as initial structure, amount of hot and cold working, time at maximum temperature, etc., were not investigated.

II. PREVIOUS INVESTIGATIONS

Excellent reviews of our knowledge concerning grain size have been given by Ward and Dorn [2] and by Shapiro [3]. The review by Ward and Dorn in particular contains an extensive bibliography.

Although these reviews reveal a wealth of information relating to the effect of austenitic grain size upon the properties of steel, very few data, aside from the effect of furnace practice, are available to indicate the effect of other variables upon the grain size, and these data are frequently contradictory. This is especially true with regard to the effect of the rate of heating.

Grossmann [4] found that different rates of heating had no effect upon the McQuaid-Ehn grain size of a 0.17-percent-carbon steel carburized at 1,700° F for 2 hours. Three samples were tested. The time of heating from room temperature to carburizing temperature was approximately 2½ hours, 30 minutes, and 5 minutes. All specimens had the same McQuaid-Ehn grain size (ASTM grain No. 5).

Digges and Jordan [5], in their investigation of the hardening characteristics of 1-percent-carbon tool steels (having "controlled" and "noncontrolled" grain sizes), studied the effect of the rate of heating upon the austenitic grain size in an indirect manner. Their work was concerned primarily with the depth of hardening obtained in ¾-inch round specimens quenched from 1,425° and 1,550° F. With all other factors constant, the depth of hardening was influenced by the grain size at the time of quenching, a coarser austenitic grain being reflected in greater depth of hardening. Variations, from 1 to 13 minutes, in the time required for the center of the ¾-inch rounds to reach the hardening temperatures had no significant influence on the depth of hardening of the two steels.

The effect of the rate of heating upon the austenitic grain size of 0.80-percent-carbon steel was studied by Tobin and Kenyon [6]. Specimens about 0.05 inch thick were heated at rates of 1.12 and 800 degrees F/min through the interval of 1,300° to 1,400° F, which included the critical temperature. They were then held at 1,400° F for 1 hour, after which all of the specimens had an ASTM No. 6 grain size. This procedure was repeated at 1,500° F. All specimens again were found to have the same grain size, which in this case was designated by Tobin and Kenyon as No. 5½. "Thus," they concluded, "the austenitic grain size of this steel was independent of the investigated rates of heating through the critical."

Herty, McBride, and Hough [7] studied the effect of the rate of heating through the transformation range on the grain size of a

0.33-percent-carbon steel deoxidized with silicon and a 0.41-percent-carbon steel deoxidized with aluminum. Half-inch cubes were heated at 1,830° F for 1 hour in order to produce a coarse-grained structure, air-cooled, and then heated through the transformation range at various rates to 1,520° F. The following data show the influence of heating rate upon the originally coarse-grained structure (3.3 grains/in.^{2} at ×100) of the "silicon-killed" steed.

Rate of heating	Grains/in.^{2} at ×100
Degrees F/min.	
2.0	368
4.0	164
5.6	25
8.5	7.6
11.0	8.4
12.6	6.4
18.0	6.0

When a ¼-inch cube of this steel in the coarse-grained condition was placed in the furnace at 1,520° F and removed 5 minutes after it had come to the temperature of the furnace, no grain refinement could be observed. The same experiments were made on the aluminum-treated steel, having initially a coarse-grained structure. At all heating rates an extremely fine grain was formed, about 200 grains per square inch at ×100.

In the present report the term "grain size" will be used to denote the grain size of the austenite established at the maximum temperatures.

III. EQUIPMENT AND PROCEDURE

To insure adequate control of the rate of heating between extremely wide limits, it was necessary to use small specimens, approximately 0.1 inch square by about 0.04 inch thick. The specimens were heated in vacuo except when a lead bath was utilized to obtain maximum rates of heating. Specimens heated in vacuo were suspended from a 32-gage Chromel-Alumel thermocouple. One wire of the thermocouple was spot-welded to the center of the flat face of the specimen, and the other wire was spot-welded to the center of the opposite face. The specimen was located within the heating chamber in the center of a Chromel heating coil, one-fourth inch in diameter by one-half inch long. This heating chamber has been described in detail by Digges [1].

As a measure of the rate of heating, the time to heat from 1,325° to 1,450° F was used. This temperature interval included the Ac_1 and Ac_3 points (1,340° and 1,425° F, respectively) in 0.50-percent-carbon steel, the material on which the first tests were made. In some of the high-carbon alloys, the Ac_{cm} point occurred at a higher temperature than 1,450° F, but it was believed that the rate of heating through this point would not be significant in comparison with the rate through the $Ac_{1\text{-}2\text{-}3}$ transformation.

In conducting the tests in vacuo, the specimens were heated rather rapidly to a temperature somewhat below 1,325° F, then at the desired rate through the range 1,325° to 1,450° F, and then again

rapidly to the required temperature. For the fastest rates of heating, specimens were plunged into a lead bath maintained at the desired temperature. With the small specimen used, an extremely rapid rate of heating (practically instantaneous) through the temperature range 1,325° to 1,450° F was obtained in the lead bath. All specimens were held at the maximum temperature for 15 minutes and then cooled in a manner suitable for outlining the austenitic grains with a proeutectoid constituent.

After preparing the specimens for microscopic examination, grain counts were made by two observers, usually at four different locations. Grain counts were made in a circle which was 7.75 square inches in area and when possible were made at a magnification of 100 diameters. Fine-grained steels were counted at higher magnifications. Since actual grain counts on steels having a decidedly mixed grain size are meaningless, in these cases the relative amounts of the predominating grain sizes were estimated according to the grain-size classification of the American Society for Testing Materials, which is given in table 1

TABLE 1.—*Grain size corresponding to American Society for Testing Materials Index Numbers*

ASTM. grain size number	Number of grains per square inch at ×100	
	Minimum	Maximum
1	¾	1½
2	1½	3
3	3	6
4	6	12
5	12	24
6	24	48
7	48	96
8	96	192

In theory the delineation of austenitic grains is simple. A steel need only be heated to the required temperature and cooled so as to reject a proeutectoid constituent to the austenitic grain boundaries which, after suitable etching, are revealed by microscopic examination at room temperature.

This procedure, however, is somewhat difficult for small specimens of hypoeutectoid steels. In order to outline the grains properly, the specimen must be cooled at exactly the right rate. Too fast a rate of cooling does not allow the precipitation of sufficient ferrite to definitely outline all the grains. A grain count on such a specimen is usually low, that is, the grains appear to be coarser than they actually are. Too slow a rate of cooling results in the precipitation of too much ferrite, accompanied by coalescence of the ferrite and obliteration of the outline of the original austenitic grains. The result of a grain count on such a specimen usually is high, that is, the grains appear to be finer than they actually are.

With hypereutectoid steels, containing at least 1 percent of carbon, it is not difficult to outline the grains at temperatures sufficiently high to dissolve the cementite completely. At the lower temperatures used for hardening these steels, however, the excess cementite tends to spheroidize and on cooling is not precipitated solely at the boundaries of the parent austenite grains.

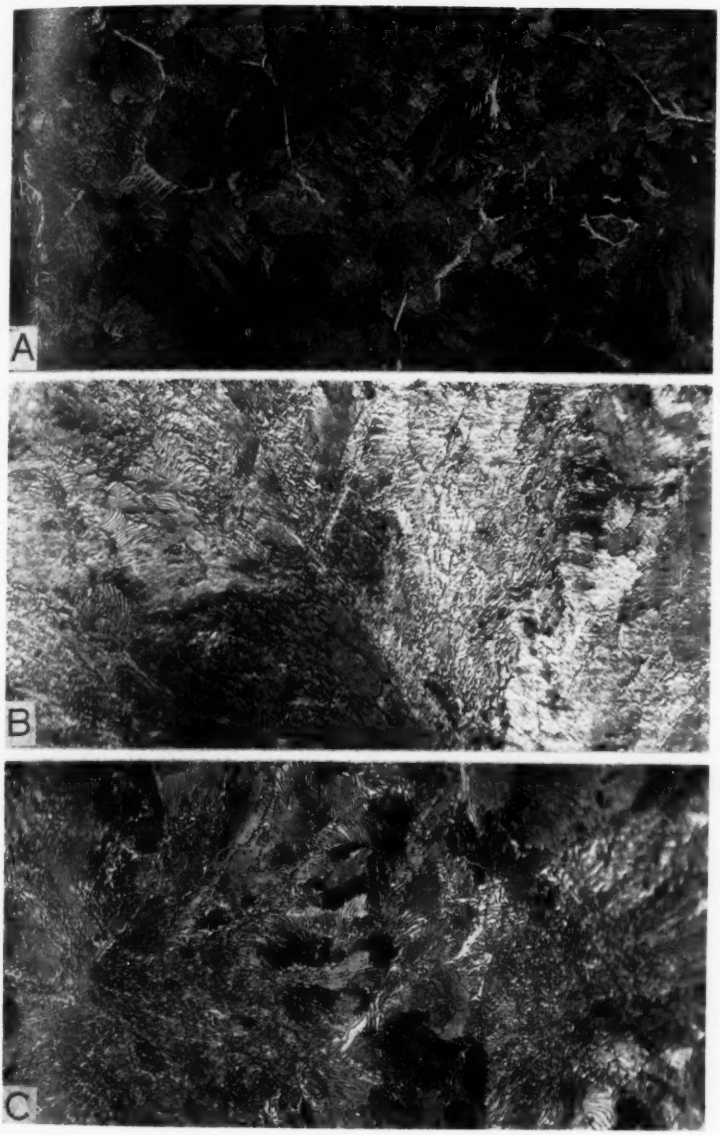

FIGURE 1.—*Initial structure of iron-carbon alloys.*

A, 0.50 percent of carbon (*HP–1*); ×100. *B*, 1.01 percent of carbon (*HP–2*); ×500. *C*, 1.21 percent of carbon (*HP–3*); ×500. Etched with 1-percent nital.

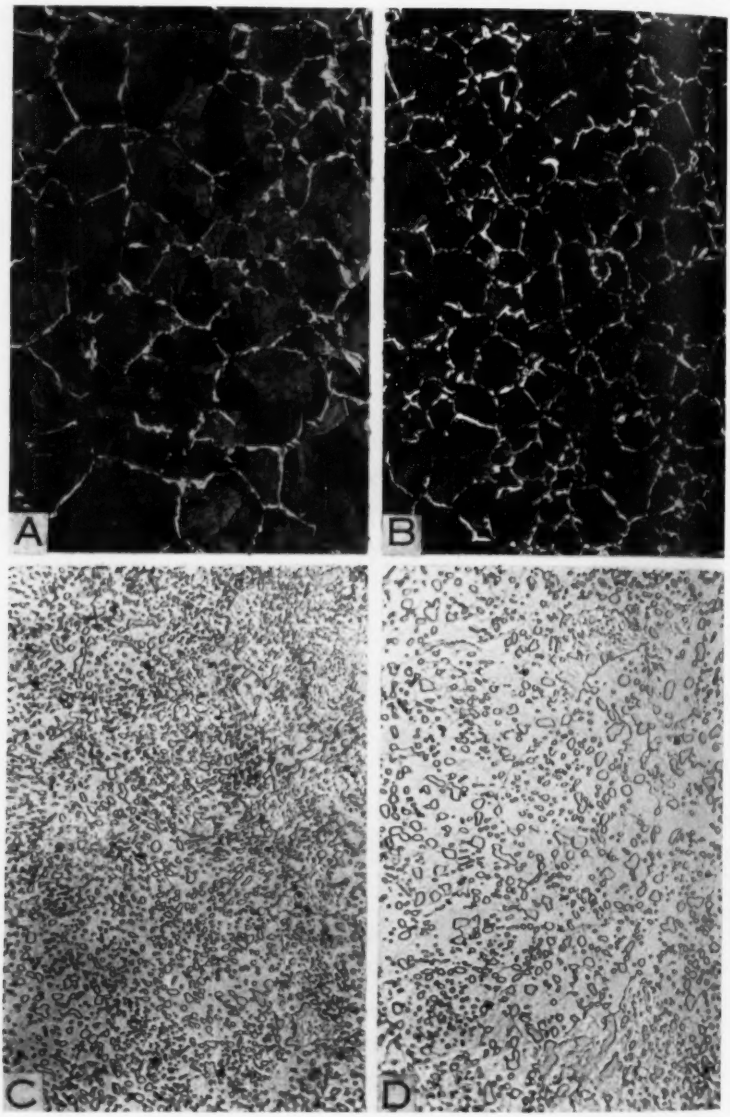

FIGURE 2.—*Initial structure of commercial plain carbon steels.*

A, 0.49 percent of carbon (*C–1*); ×100. *B*, 0.49 percent of carbon (*C–2*); ×100. *C*, 1.05 percent of carbon (*C–3*); ×500. *D*, 1.06 percent of carbon (*C–4*); ×500. Etched with 1-percent nital.

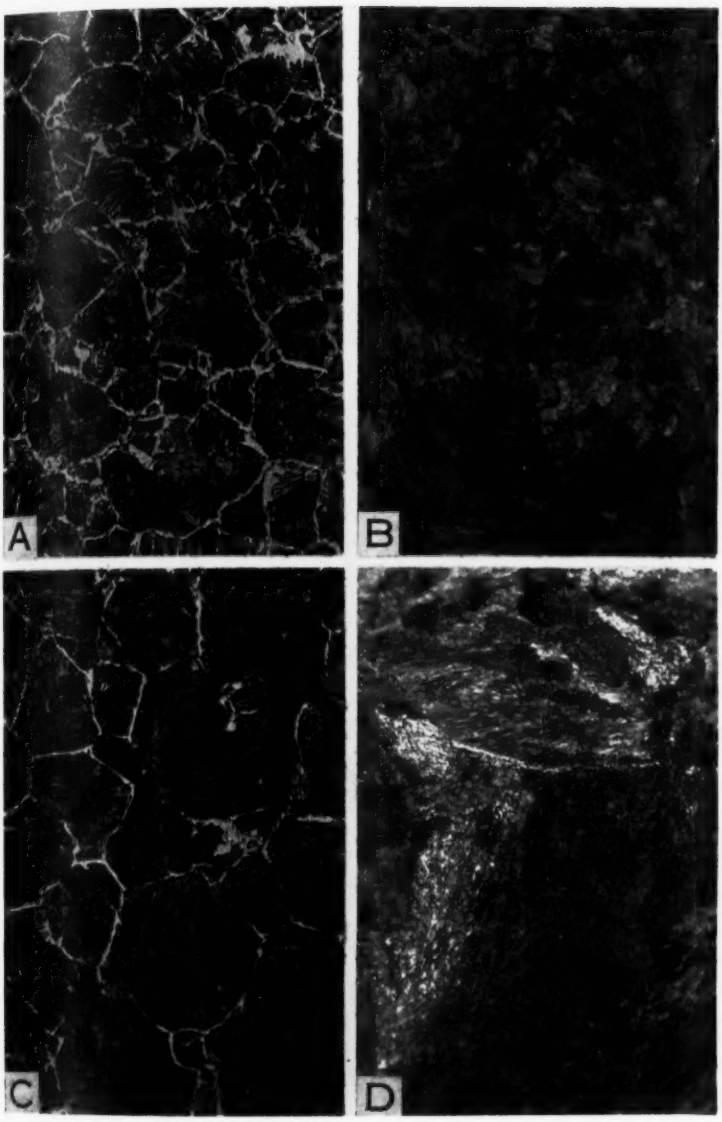

FIGURE 3.—*Initial structure of experimental plain carbon steels.*

A, 0.48 percent of carbon (*PC–1*); ×100. *B*, 1.12 percent of carbon (*PC–2*); ×500. *C*, 0.57 percent of carbon (*GC–1*); ×100. *D*, 1.02 percent of carbon (*GC–2*); ×500. Etched with 1-percent nital.

IV. MATERIALS

1. HIGH-PURITY ALLOYS OF IRON AND CARBON

The iron-carbon alloys used in the present study were the same as those used in previous investigations, and their preparation has been described in a previous report [8]. Essentially, the preparation consisted in carburizing hot- and cold-worked specimens of vacuum-melted electrolytic iron in a mixture of hydrogen and benzene vapor, and subsequently homogenizing by heating in vacuo at 1,700° F. Cooling from the temperature used in homogenizing was sufficiently rapid to produce sorbite (fine pearlite) in all the alloys. Specimens used for the determination of grain size had an initial structure of sorbite, as shown in figure 1.

Results of spectrochemical and chemical analyses and the determination of the gas content by the vacuum-fusion method showed that sulfur, nickel, cobalt, and oxygen were the major impurities in the alloys. These elements amounted to about 0.021 percent, whereas the total percentage (by weight) of all impurities determined was about 0.030. The compositions of the alloys are given in table 2.

2. COMMERCIAL PLAIN CARBON STEELS

Commercial plain carbon steels containing approximately 0.5 and 1 percent of carbon were used. Two heats containing 0.5 percent of carbon were furnished through the courtesy of a large producer, and two heats of tool steel containing 1 percent of carbon were purchased through regular channels of supply. As shown in table 2, the most significant difference in the chemical compositions of the hypoeutectoid steels was in the content of aluminum and of aluminum oxide. Similarly, the hypereutectoid steels differed in the proportions of aluminum and aluminum oxide. The heats containing appreciably higher proportions of metallic aluminum (that soluble in acid) and aluminum oxide were produced under conditions intended to control the austenitic grain size; the heats of relatively low total aluminum content were produced under conditions which resulted in noncontrol of austenitic grain size. All heats were representative of good commercial open-hearth practice. For the determination of grain size, specimens of the hypoeutectoid steels were used in the hot-rolled condition, with an initial structure of sorbite and ferrite (fig. 2, *A* and *B*). The hypereutectoid steels were annealed to produce an initial structure of spheroidized cementite (fig. 2, *C* and *D*).

3. EXPERIMENTAL PLAIN CARBON STEELS

Two series of carbon steels were prepared from specimens cut from a forging of a plain carbon steel, which contained 0.20 percent of carbon and appreciable amounts of aluminum. The first series was prepared by pack-carburizing these specimens in a mixture of approximately 85 percent of wood charcoal and 15 percent of barium carbonate at 1,700° F for various periods of time. The carburized specimens were homogenized by pack-annealing at 1,700° F for 4 to 5 hours before normalizing from the usually recommended temperature range. The second series was prepared by carburizing specimens in hydrogen-benzene vapor at 1,700° F and subsequently homogenizing

TABLE 2.—*Chemical composition (percentage by weight) of materials used*

Identification	C	Mn	P	S	Si	Cr	Ni	Co	Mo	V	Al_2O_3	Acid-soluble Al	Total Al	H_2	O_2	N_2
HIGH-PURITY ALLOYS OF IRON AND CARBON																
HP-1	0.50	0.002	<0.001	0.004	0.002	.....	0.007	0.007	.....	.....	.....	.....	.....	0.0002	0.003	0.001
HP-2	1.01	.002	<.001	.004	.002	.....	.007	.007	.....	.....	.....	.....	.....	.0002	.003	.001
HP-3	1.21	.002	<.001	.004	.002	.....	.007	.007	.....	.....	.....	.....	.....	.0002	.003	.001
COMMERCIAL STEELS																
C-1	0.49	0.79	0.023	0.023	0.22	.....	.....	.....	.....	.....	0.001	0.002	0.0025	0.0002	0.004	0.004
C-2	.49	.80	.022	.028	.21	.....	.....	.....	.....	.....	.007	.016	.0195	.0002	.003	.004
C-3	1.05	.28	.019	.013	.22	0.08	0.13	.....	<0.01	<0.01	.006	.001	.004	.0002	.007	.009
C-4	1.06	.26	.013	.011	.19	.09	.10	.....	<.01	<.01	.010	.008	.013	.0003	.009	.007
EXPERIMENTAL STEELS (PACK-CARBURIZED)																
PC-1	0.48	0.46	0.043	0.025	0.15	0.02	0.011	.....	.....	Trace	0.017	0.006	0.015	.....	.....	.....
PC-2	1.12	.46	.043	.025	.15	.02	.011	.....	.....	Trace	.017	.006	.015	.....	.....	.....
EXPERIMENTAL STEELS (GAS-CARBURIZED)																
GC-1	0.57	0.46	0.043	0.025	0.15	0.02	0.011	.....	.....	Trace	0.017	0.006	0.015	.....	.....	.....
GC-2	1.02	.46	.043	.025	.15	.02	.011	.....	.....	Trace	.017	.006	.015	.....	.....	.....

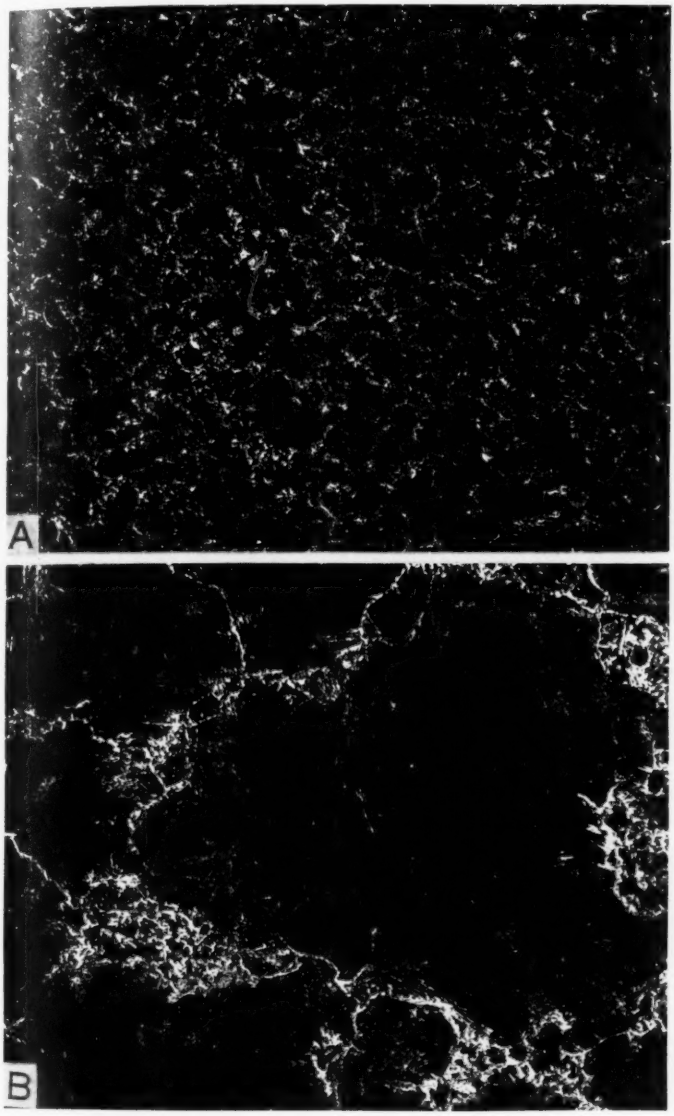

FIGURE 4.—*Grain size at 1,475° F of iron-carbon alloy HP–1 (0.50 percent of carbon).*

A, Heated in lead bath; estimated ASTM grain No. 7. *B*, Heated 7.7 degrees F per minute; estimated ASTM grain No. −2 to 1; grains not outlined in approximately 20 percent of the area, which consisted of coalesced ferrite and pearlite. Etched with 1-percent nital. ×100.

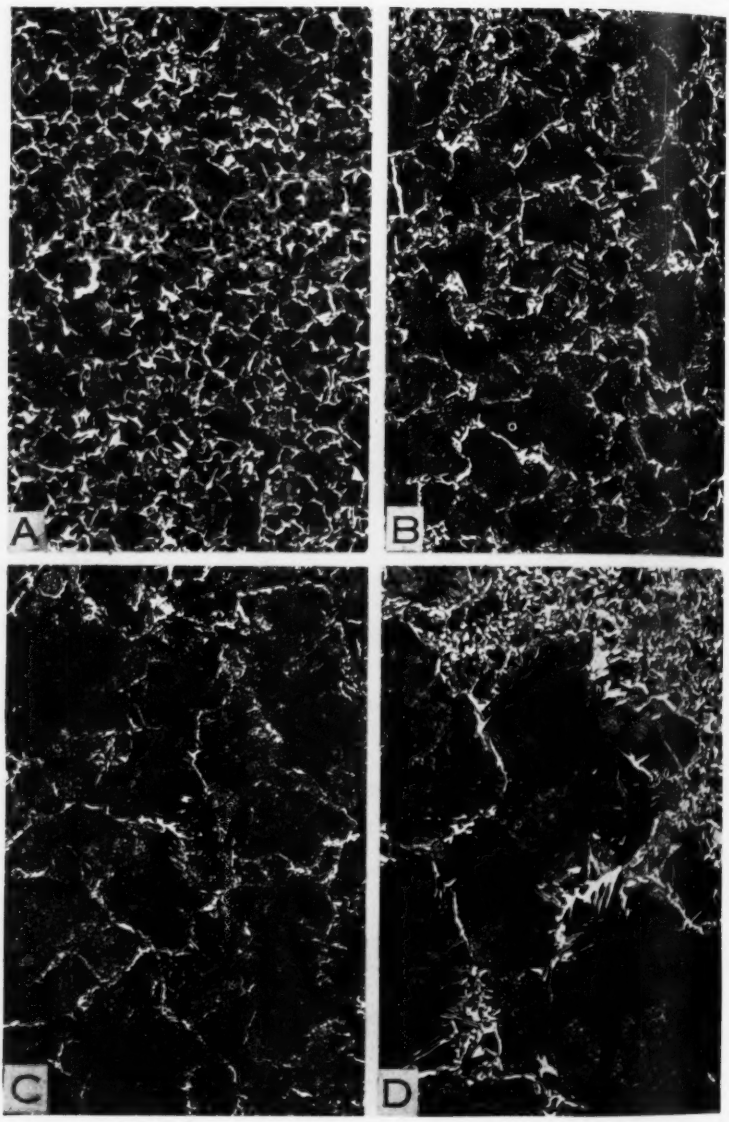

FIGURE 5.—*Grain size at 1,500° F of iron-carbon alloy HP–1 (0.50 percent of carbon).*

A, Heated in lead bath; 35 grains per square inch at ×100. *B*, Heated 440 degrees F per minute; 12 grains per square inch at ×100. *C*, Heated 155 degrees F per minute; 4.4 grains per square inch at ×100. *D*, Heated 13 degrees F per minute; estimated ASTM grain No. 1 or larger; grains not outlined in approximately 30 percent of the area, which consisted of coalesced ferrite and pearlite. Etched with 1-percent nital. ×100.

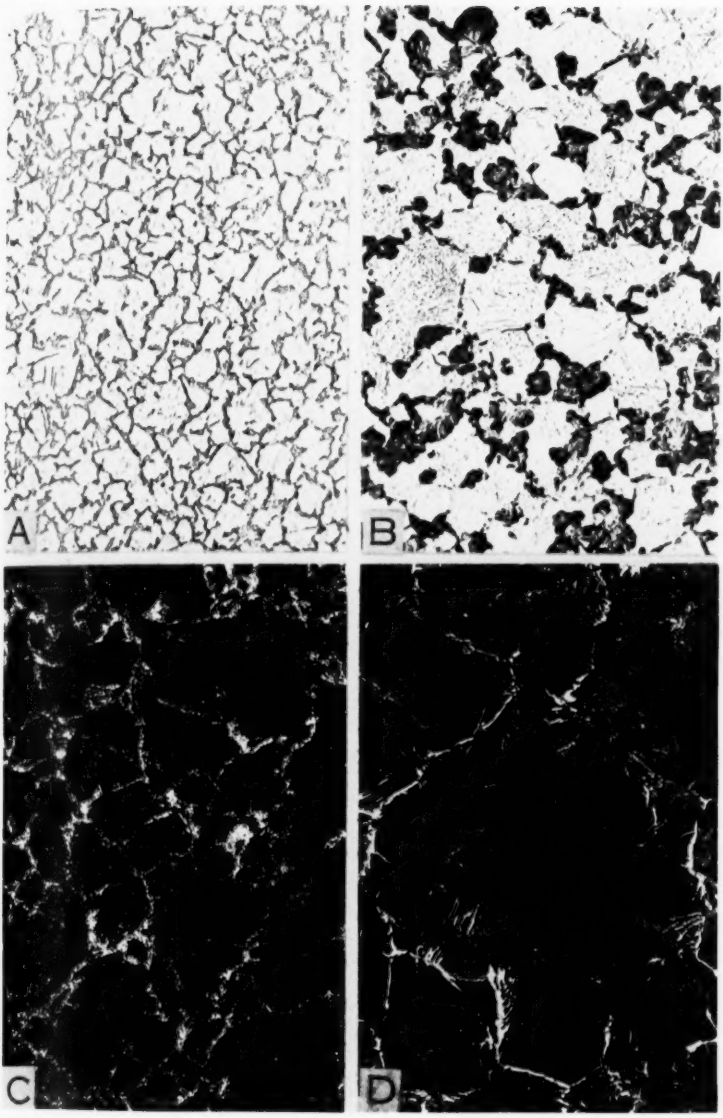

FIGURE 6.—*Grain size at 1,600° F of iron-carbon alloy HP–1 (0.50 percent of carbon).*

A, Heated in lead bath; 50 grains per square inch at ×100. *B*, Heated 940 degrees F per minute; 12 grains per square inch at ×100. *C*, Heated 280 degrees F per minute; 7.3 grains per square inch at ×100. *D*, Heated 3 degrees F per minute; 1 grain per square inch at ×100. Etched with 1-percent nital. ×100.

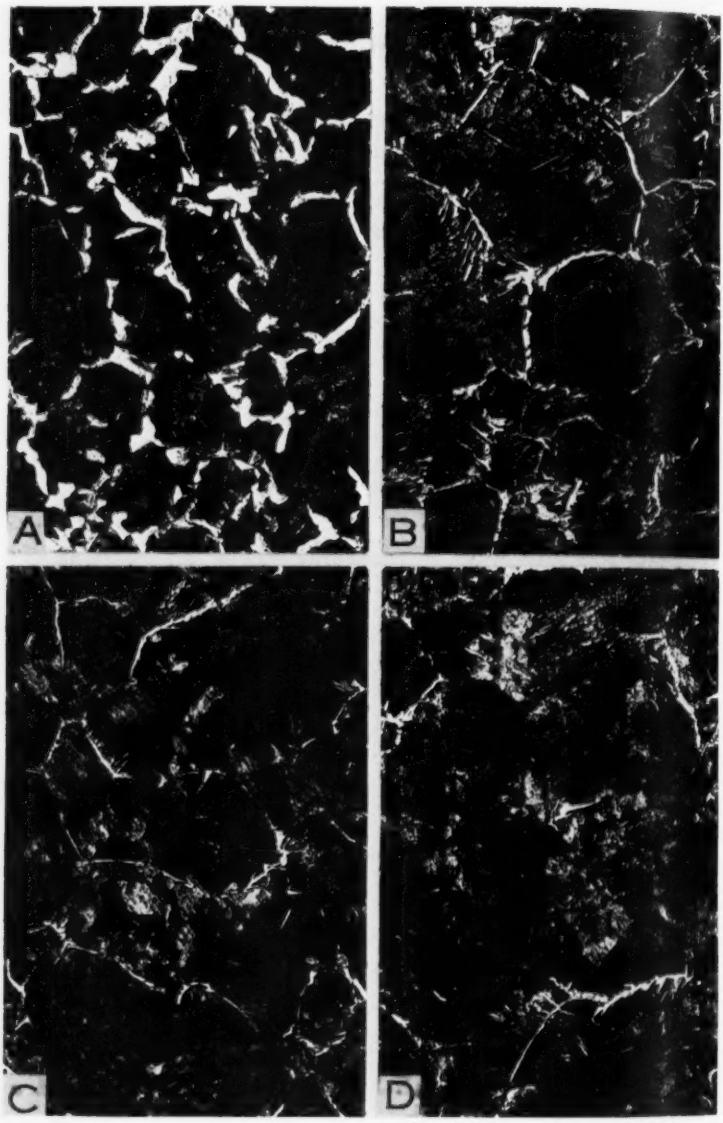

FIGURE 7.—*Grain size at 1,800° F of iron-carbon alloy HP–1 (0.50 percent of carbon).*

A, Heated in lead bath; 5.4 grains per square inch at ×100. *B*, Heated 575 degrees F per minute; 3.3 grains per square inch at ×100. *C*, Heated 11 degrees F per minute; estimated ASTM grain No. 2. *D*, Heated 3 degrees F per minute; 1 grain per square inch at ×100. Etched with 1-percent nital. ×100.

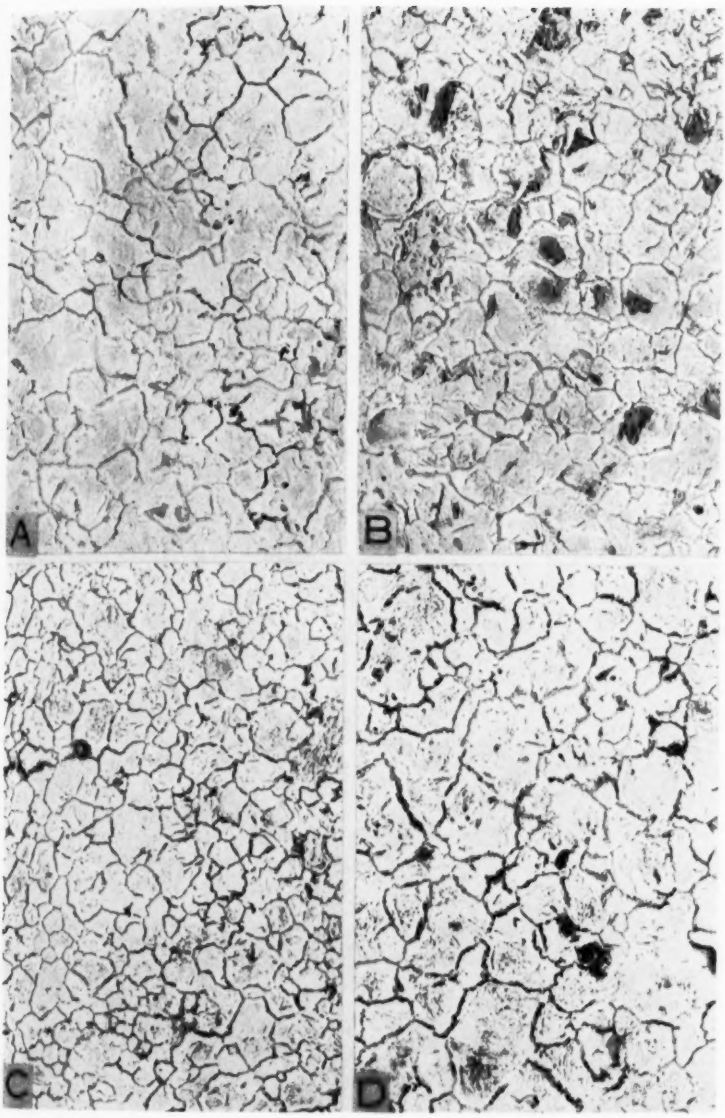

FIGURE 8.—*Grain size at 1,600° F of iron-carbon alloy HP-2 (1.01 percent of carbon).*

A, Heated in lead bath; 20 grains per square inch at ×100. *B*, Heated 625 degrees F per minute; 15 grains per square inch at ×100. *C*, Heated 625 degrees F per minute; 40 grains per square inch at ×100. *D*, Heated 13 degrees F per minute; 11 grains per square inch at ×100. Etched with boiling sodium-picrate solution. ×100.

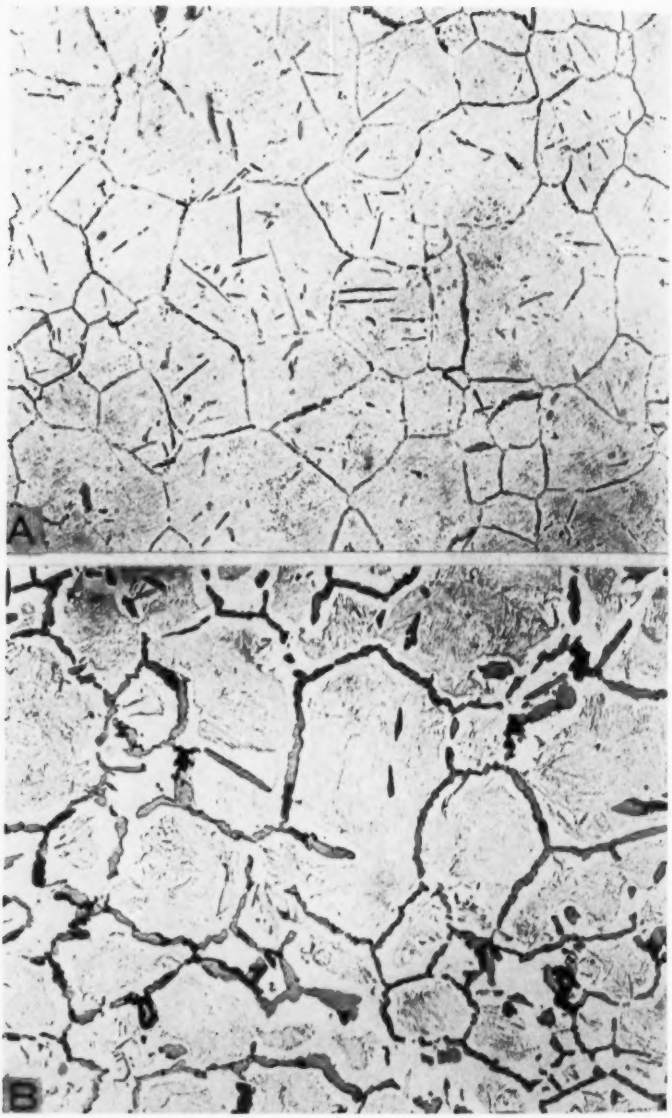

FIGURE 9.—*Grain size at 1,800° F of iron-carbon alloy HP–3 (1.21 percent of carbon).*

A, Heated in lead bath; 5.3 grains per square inch at ×100. *B*, Heated 3 degrees F per minute; 2.4 grains per square inch at ×100. Etched with boiling sodium picrate solution. ×100.

by heating in vacuo at 1,700° F. The hypoeutectoid steels had an initial structure of sorbite and ferrite (fig. 3, *A* and *C*), and the hypereutectoid steels had an initial structure of sorbite (fig. 3, *B* and *D*). The compositions of these steels are given in table 2.

V. RESULTS

1. HIGH-PURITY ALLOYS OF IRON AND CARBON

In this group of alloys a comprehensive series of tests was made on the alloy containing 0.50 percent of carbon (*HP–1*). The grain size of this alloy at 1,475°, 1,500°, 1,600°, and 1,800° F was determined for widely varying rates of heating through the transformation range. The grain size of the alloy containing 1.01 percent of carbon (*HP–2*) was determined only at 1,600° F and that of the alloy containing 1.21 percent of carbon (*HP–3*) was determined only at 1,800° F, with various rates of heating. The results are summarized in tables 3, 4, and 5. Representative micrographs are reproduced in figures 4 to 9, inclusive.

TABLE 3.—*Grain size of iron-carbon alloy HP–1 (0.50 percent of carbon)*

Temperature	Rate of heating between 1,325° and 1,450° F	Grains/in.² at ×100	Remarks
°F	*Degrees F/min.*		
1,475	(1)	.....	Estimated ASTM grain No. 7.
1,475	7.7	.....	Estimated ASTM grain No. −2 to 1. Grains not outlined in approximately 20 percent of the area, which consisted of coalesced ferrite and pearlite.
1,500	(1)	35	
1,500	440	12	
1,500	200	4.3	
1,500	190	2.5	
1,500	170	3.5	
1,500	155	4.4	
1,500	13	.....	Estimated ASTM grain No. 1 or larger. Grains not outlined in approximately 30 percent of the area, which consisted of coalesced ferrite and pearlite.
1,500	3	.....	Estimated ASTM grain No. 1 or larger. Grains not outlined in approximately 70 per cent of the area, which consisted of coalesced ferrite and pearlite.
1,600	(1)	50	
1,600	940	12	
1,600	470	9	
1,600	325	9	
1,600	280	7.3	
1,600	100	1.2	
1,600	80	2.5	
1,600	10	0.4	
1,600	3	1.0	
1,800	(1)	5.4	
1,800	1,875	3.0	
1,800	1,875	2.3	
1,800	1,365	5.2	
1,800	575	3.3	
1,800	415	4.9	
1,800	12	0.6	
1,800	11	.....	Estimated ASTM grain No. 2.
1,800	3	1.0	

(1) Heated very rapidly in lead bath.

TABLE 4.—*Grain size at 1,600° F of iron-carbon alloy HP–2 (1.01 percent of carbon)*

Rate of Heating between 1,325° and 1,450° F	Grains/in.[2] at ×100	Remarks
Degrees F/min		
(1)	20	
(1)	20	
1,070	.....	Mixed grains—estimated 10 percent of ASTM grain No. 3 to 4 and 90 percent of No. 7 to 8.
830	.....	Mixed grains—estimated 10 percent of ASTM grain No. 3 to 4 and 90 percent of No. 7 to 8.
625	12	
625	11	
625	15	
625	40	
580	12	
440	.....	Mixed—noted all grains from ASTM grain No. 2 to 7. Few No. 2 to 3, majority No. 5.
420	10	
400	.....	Estimated ASTM grain No. 4 to 5.
13	11	

[1] Heated very rapidly in lead bath.

TABLE 5.—*Grain size at 1,800 °F of iron-carbon alloy HP–3 (1.21 percent of carbon)*

Rate of Heating between 1,325° and 1,450° F	Grains/in.[2] at ×100
Degrees F/min	
(1)	5.3
1,875	3.2
1,670	2.8
1,670	2.7
1,670	3.1
1,360	3.7
1,250	2.0
1,250	2.3
1,150	3.5
3	2.4
1.4	3.0

[1] Heated very rapidly in lead bath.

With the 0.50-percent-carbon alloy, the rate of heating through the transformation range had a very marked effect upon the grain size at all temperatures investigated below 1,800° F. The grain size increased with decrease in rate of heating. This trend was most pronounced in the upper range of heating rates—that is, after the rate of heating had decreased to a certain value, further decrease in rate caused but little increase in grain size of this alloy. At 1,800° F, coarse grains (ASTM grain No. 1 to 3) were produced regardless of the rate of heating through the transformation range. It should be pointed out, however, that the trend of increase in grain size with decrease in rate of heating also existed for the specimens heated at 1,800° F.

With slow rates of heating, coarse grains of the same order of magnitude were obtained at all temperatures. However, considerably coarser grains were produced at 1,475°, 1,500°, and 1,600° F with slow rates of heating than at 1,800° F with fast rates of heating.

The coarse grains produced at 1,475° and 1,500° F in the 0.50-percent-carbon alloy with relatively slow rates of heating were found, upon cooling, to be associated with areas containing numerous ferrite and pearlite islands (figs. 4, *B*, and 5, *D*). The appearance, however, suggested that these were originally not mixed austenitic grains but that some of the large grains of the parent austenite had broken into many small grains of ferrite and pearlite. At 1,500° F, this condition was noticeable for the specimen heated at the rate of 13 degrees F/min (table 3 and fig. 5, *D*); for the specimen heated at the still slower rate of 3 degrees F/min, this condition was considerably more marked. With slow rates of heating, specimens heated to 1,600 and 1,800° F were also coarse grained, but coalescence of ferrite and pearlite was not observed.

The results obtained for the 1.01-percent-carbon alloy at 1,600° F (table 4 and fig. 8) showed that the finest grains were not obtained with the most rapid heating. For example, approximately 20 grains per square inch at 100 diameters (ASTM grain No. 5) were produced in the specimens heated rapidly in a lead bath, whereas mixed grains, estimated as 10 percent of ASTM grain No. 3 to 4 and 90 percent of No. 7 to 8 (not shown in fig. 8), were produced by decreasing the heating rate to about 1,000 degrees F/min. Furthermore, there was an appreciable scatter in values of the grain sizes obtained when the rates were approximately constant and of the order of 625 degrees F/min (fig. 8, *B* and *C*). These results suggest the possibility of the existence of a critical rate of heating through the transformation range that produces austenitic grains of minimum size. That is, larger grains may be produced when the heating rate is either faster or slower than the critical value.

The rate of heating through the transformation range had little effect on the grain size, established at 1,800° F, of the iron-carbon alloy containing 1.21 percent of carbon (table 5 and fig. 9).

The final austenitic grain size of these high-purity alloys of iron and carbon was influenced by the rate of heating through the transformation range and by the maximum temperature to which the alloys were heated.

2. COMMERCIAL PLAIN CARBON STEELS

It was believed that the general relation between rate of heating and grain size of commercial steels could be established by securing data on the effect of rate of heating on two hypoeutectoid and two hypereutectoid steels, one of which in each case had been treated to produce what is usually termed a "controlled" steel. The results obtained are summarized in tables 6, 7, 8, and 9. Representative micrographs of the grain sizes at different temperatures with various rates of heating are shown in figures 10 to 13, inclusive.

TABLE 6.—*Grain size of steel C–1 (0.49 percent of carbon, noncontrolled)*

Temperature	Rate of heating between 1,325° and 1,450° F	Grains/in.² at × 100	Remarks
°F	*Degrees F/min*		
1,475	(1)	35	
1,475	(1)	45	
1,475	(1)	40	
1,475	(1)	45	
1,475	(1)	55	
1,475	185	----------	Estimated 40 percent of ASTM grain No. 5 to 6; 60 percent No. 8.
1,475	105	110	
1,475	3.5	120	
1,475	3.1	105	
1,600	(1)	30	
1,600	(1)	30	
1,600	160	35	
1,600	4	45	
1,800	(1)	6.1	
1,800	(1)	3.5	
1,800	22	4.5	
1,800	14	7.0	
1,800	12	4.9	
1,800	2.8	9.0	

[1] Heated very rapidly in lead bath.

TABLE 7.—*Grain size of steel C–2 (0.49 percent of carbon, controlled)*

Temperature	Rate of heating between 1,325° and 1,450° F	Grains/in.² at × 100	Remarks
°F	*Degrees F/min*		
1,475	(1)	----------	Estimated 30 percent of ASTM grain No. 5, 70 percent of No. 8.
1,475	(1)	----------	Estimated 60 percent of ASTM grain No. 5 to 6, 40 percent of No. 8.
1,475	(1)	75	
1,475	(1)	80	
1,475	(1)	80	
1,475	41	----------	Estimated 10 percent of ASTM grain No. 7, 90 percent of No. 8.
1,475	12	----------	Estimated 15 percent of ASTM grain No. 6 to 7, 85 percent of No. 8.
1,475	2.7	120	
1,000	(1)	55	
1,600	(1)	90	
1,600	13	----------	Estimated 10 percent of ASTM grain No. 6, 90 percent of No. 8.
1,600	2.7	150	
1,600	2.7	150	
1,800	(1)	25	
1,800	(1)	23	
1,800	11	80	
1,800	3.9	70	

[1] Heated very rapidly in lead bath.

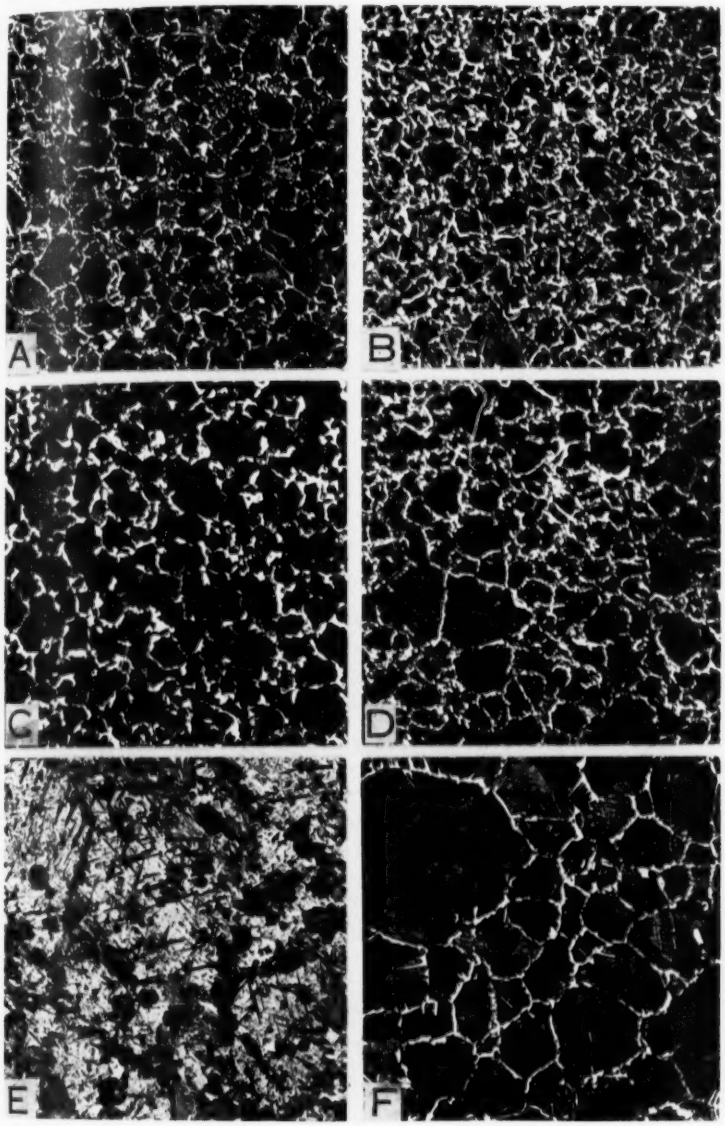

FIGURE 10.—*Grain size at different temperatures of commercial steel C-1 (0.49 percent of carbon, noncontrolled grain size).*

A, 1,475° F; heated in lead bath; 35 grains per square inch at ×100. *B*, 1,475° F; heated 3.1 degrees F per minute; 105 grains per square inch at ×100. *C*, 1,600° F; heated in lead bath; 30 grains per square inch at ×100. *D*, 1,600° F; heated 4 degrees F per minute; 45 grains per square inch at ×100. *E*, 1,800° F; heated in lead bath; 6.1 grains per square inch at ×100. *F*, 1,800° F; heated 2.8 degrees F per minute; 9 grains per square inch at ×100. Etched with 1-percent nital. ×100.

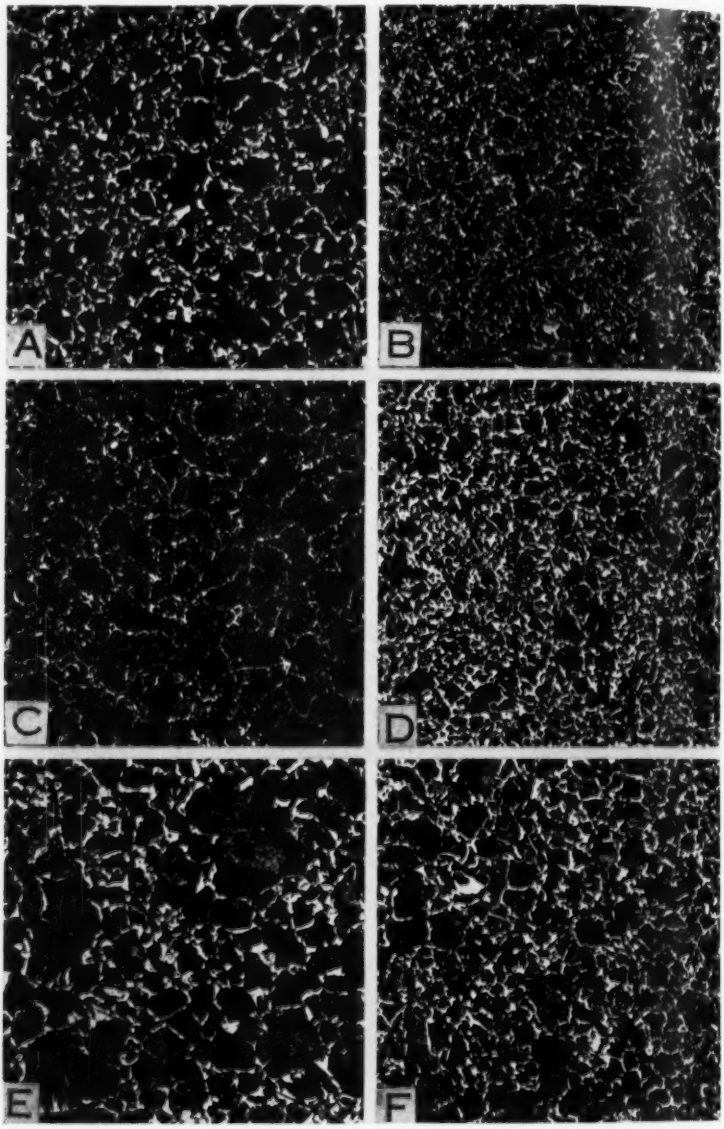

FIGURE 11.—*Grain size at different temperatures of commercial steel C–2 (0.49 percent of carbon, controlled grain size).*

A, 1,475° F; heated in lead bath; 80 grains per square inch at ×100. *B*, 1,475° F; heated 2.7 degrees F per minute; 120 grains per square inch at ×100. *C*, 1,600° F; heated in lead bath; 55 grains per square inch at ×100. *D*, 1,600° F; heated 2.7 degrees F per minute; 150 grains per square inch at ×100. *E*, 1,800° F; heated in lead bath; 25 grains per square inch at ×100. *F*, 1,800° F; heated 3.9 degrees F per minute; 70 grains per square inch at ×100. Etched with 1-percent nital. ×100.

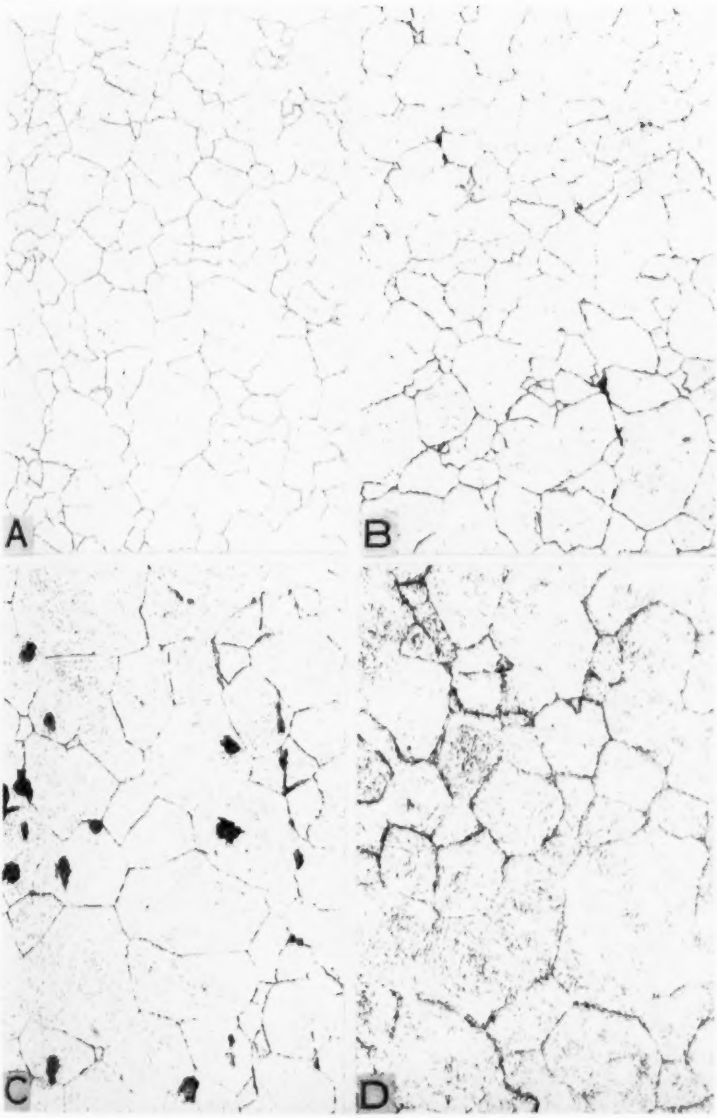

FIGURE 12.—*Grain size at different temperatures of commercial steel C–3 (1.05 percent of carbon, noncontrolled grain size).*

A, 1,650° F; heated in lead bath; 20 grains per square inch at ×100. *B*, 1,650° F; heated 3 degrees F per minute; 16 grains per square inch at ×100. *C*, 1,800° F; heated in lead bath; 5.3 grains per square inch at ×100. *D*, 1,800° F; heated 3.6 degrees F per minute; 5.9 grains per square inch at ×100. Etched with 1-percent nital. ×100.

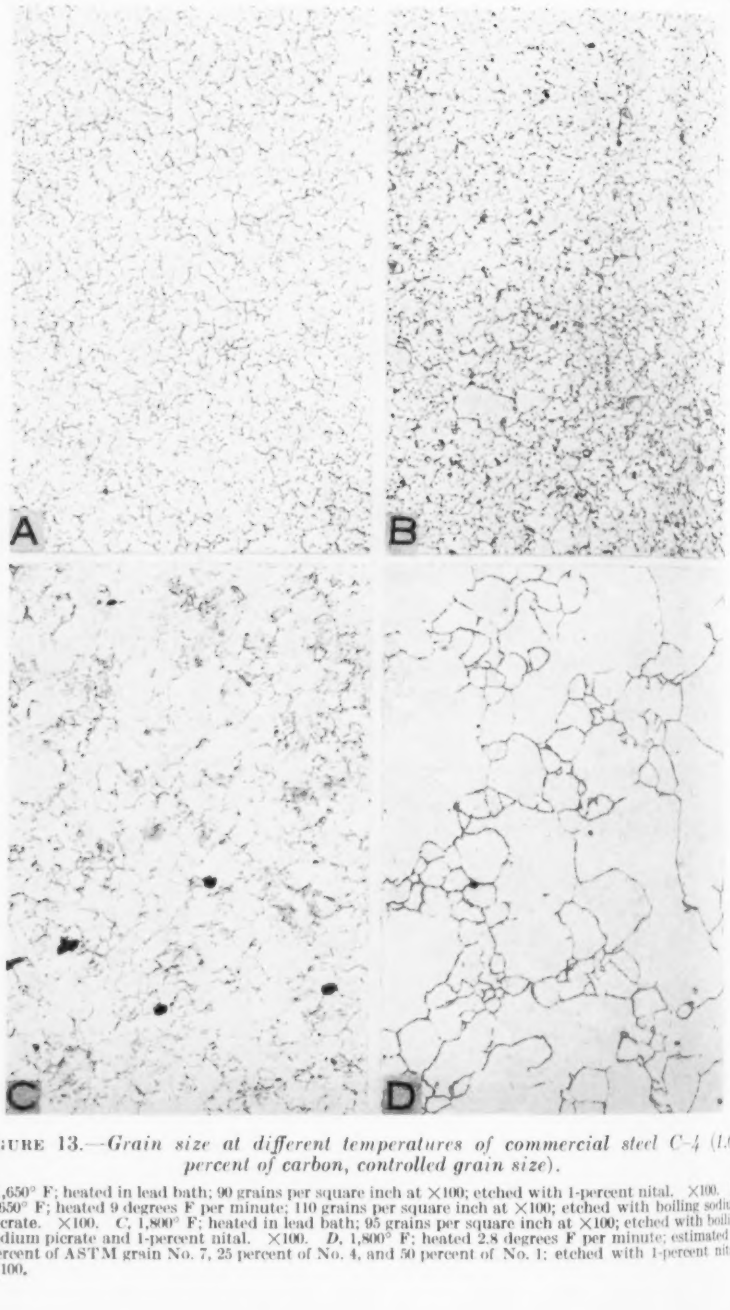

FIGURE 13.—*Grain size at different temperatures of commercial steel C–4 (1.09 percent of carbon, controlled grain size).*

A, 1,650° F; heated in lead bath; 90 grains per square inch at ×100; etched with 1-percent nital. ×100. *B*, 1,650° F; heated 9 degrees F per minute; 110 grains per square inch at ×100; etched with boiling sodium picrate. ×100. *C*, 1,800° F; heated in lead bath; 95 grains per square inch at ×100; etched with boiling sodium picrate and 1-percent nital. ×100. *D*, 1,800° F; heated 2.8 degrees F per minute; estimated 25 percent of ASTM grain No. 7, 25 percent of No. 4, and 50 percent of No. 1; etched with 1-percent nital. ×100.

TABLE 8.—*Grain size of steel C–3 (1.05 percent of carbon, noncontrolled)*

Temperature	Rate of heating between 1,325° and 1,450° F	Grains/in.² at ×100
°F	*Degrees F/min*	
1,650	(1)	20
1,650	750	26
1,650	9	18
1,650	3	16
1,800	(1)	5.3
1,800	(1)	4.9
1,800	9	5.2
1,800	3.6	5.9

1 Heated very rapidly in lead bath.

TABLE 9.—*Grain size of steel C–4 (1.06 percent of carbon, controlled)*

Temperature	Rate of heating between 1,325° and 1,450° F	Grains/in.² at ×100	Remarks
°F	*Degrees F/min*		
1,650	(1)	90	
1,650	(1)	85	
1,650	940	40	
1,650	9	110	
1,800	(1)	95	
1,800	(1)	95	
1,800	9	13	
1,800	8.6	----------	Estimated 25 percent of ASTM grain No. 6 to 8, 15 percent of No. 4 to 5, 60 percent of No. 1 or larger.
1,800	2.8	----------	Estimated 25 percent of ASTM grain No. 7, 25 percent of No. 4, 50 percent of No. 1.

1 Heated very rapidly in lead bath.

The rate of heating through the transformation range influenced the grain size established at 1,475° F (recommended hardening temperature) of the hypoeutectoid steels with both "noncontrolled" and "controlled" grain size (Figs. 10, *A* and *B*, and 11, *A* and *B*) and at 1,600° and 1,800° F of the controlled steel (Fig. 11, *C*, *D*, *E*, and *F*). The rate of heating had no appreciable effect on the grain size at 1,600° and 1,800° F of the noncontrolled steel (Fig. 10, *C*, *D*, *E*, and *F*). Regardless of the rate of heating, both of these steels were fine-grained at 1,475° F, and the controlled steel was fine-grained at 1,600° F.

Under conditions in which the grain size was influenced by the rate of heating these commercial 0.49-percent-carbon steels, the minimum-sized grains were not obtained with the maximum rate of heating. That is, in these steels the grain size tended to *increase* with *increase* in rate of heating. This trend is contrary to that observed in the high-purity alloy of iron and carbon having a similar carbon content.

The rate of heating had no significant effect on the grain size at 1,650° and 1,800° F of the noncontrolled steel containing 1.05 percent of carbon (table 8 and fig. 12). With the controlled steel, the rate of heating influenced the grain size at both 1,650° and 1,800° F. At the lower temperature, the coarsest grains were obtained with an intermediate rate of heating (table 9). Slow rates of heating pro-

duced grains of the same order of magnitude as very rapid rates (fig. 13, *A* and *B*). At 1,800° F, the grain size of this steel increased with decrease in rate of heating (fig. 13, *C* and *D*), a trend similar to that shown by the 0.50-percent-carbon alloy and contrary to that of the commercial hypoeutectoid steels.

These results obtained with commercial plain carbon steels, both noncontrolled and controlled, show that in certain heats the austenitic grain sizes established at various temperatures were sensitive to the rate of heating through the transformation range. In other heats, the grain sizes were insensitive to the rate of heating. The cause of the observed variations cannot be definitely stated at this time.

3. EXPERIMENTAL PLAIN CARBON STEELS

It is apparent that the effect of rate of heating on the grain size of the iron-carbon alloys was different from the effect on commercial plain carbon steels. Since the preparation of the iron-carbon alloys was different from that of the commercial steels, it was thought that the procedure used in preparing the alloys might have had some influence on the relation between the heating rate and grain size. Specimens were available of both hypoeutectoid and hypereutectoid steels prepared from a 0.2-percent-carbon steel (aluminum-killed) by pack-carburizing and by gas-carburizing, as described in a previous section of this report. If the method of introduction of carbon (gas-carburizing) were responsible for the differences observed in the grain sizes of the iron-carbon alloys with various rates of heating, it was thought that perhaps a similar relation would be shown in the carburized steels.

The grain size established at 1,600° F with various rates of heating was determined for the experimental steels containing about 0.5 and 1 percent of carbon. The results of the grain-size measurements are summarized in table 10. These data and the representative micrographs (figs. 14 and 15) show that the rate of heating had no significant effect on the grain size at 1,600° F of any of these steels.

The results obtained with the experimental steels indicate that the variations in grain size with rate of heating of the high-purity alloys of iron and carbon cannot be attributed solely to the method by which carbon was introduced.

TABLE 10.—*Grain size at 1,600° F of experimental steels*

Steel	Rate of heating between 1,325° and 1,450 °F	Grains/in.[2] at ×100	Remarks
	Degrees F/min.		
PC-1 (0.48% C)	(1)	45	Estimated 15 percent of ASTM grain No. 2 to 3, 15 percent of No. 4, 70 percent of No. 7 to 8.
Do	10		
PC-2 (1.12% C)	(1)	15	Few ASTM grain No. 2 to 3 and 7.
Do	(1)	16	Do.
Do	17		Estimated ASTM grain No. 4.
Do	17	16	
GC-1 (0.57% C)	(1)		Estimated about 60 percent of ASTM grain No. 2 to 4, 40 percent of No. 7 to 8.
Do	11		Estimated 50 percent ASTM grain No. 2 to 4, 50 percent of No. 7 to 8.
GC-2 (1.02% C)	(1)	14	
Do	(1)	14	
Do	7	11	

[1] Heated very rapidly in lead bath.

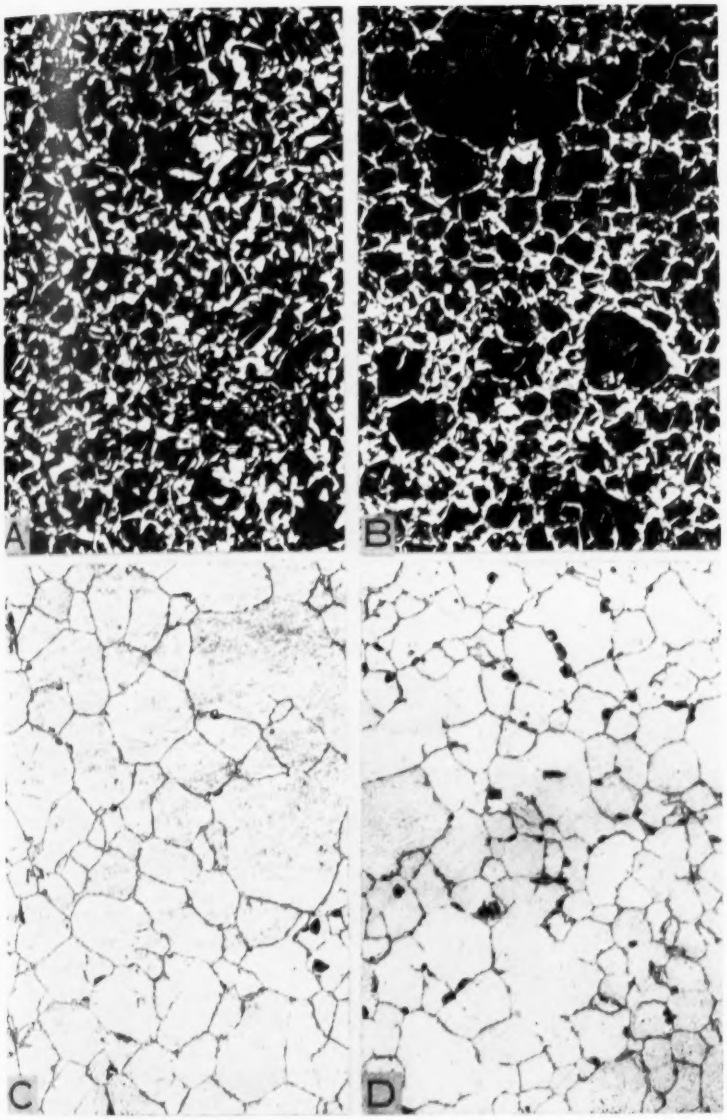

FIGURE 14.—*Grain size at 1,600° F of experimental plain carbon steels (pack carburized).*

A, *PC-1* (0.48 percent of carbon); heated in lead bath; 45 grains per square inch at ×100. *B*, *PC-1* (0.48 percent of carbon); heated 10 degrees F per minute; estimated 15 percent of ASTM grain No. 2 to 3, 15 percent of No. 4, and 70 percent of No. 7 to 8. *C*, *PC-2* (1.12 percent of carbon); heated in lead bath. 15 grains per square inch at ×100. *D*, *PC-2* (1.12 percent of carbon); heated 17 degrees F per minute; 16 grains per square inch at ×100. Etched with 1-percent nital. ×100.

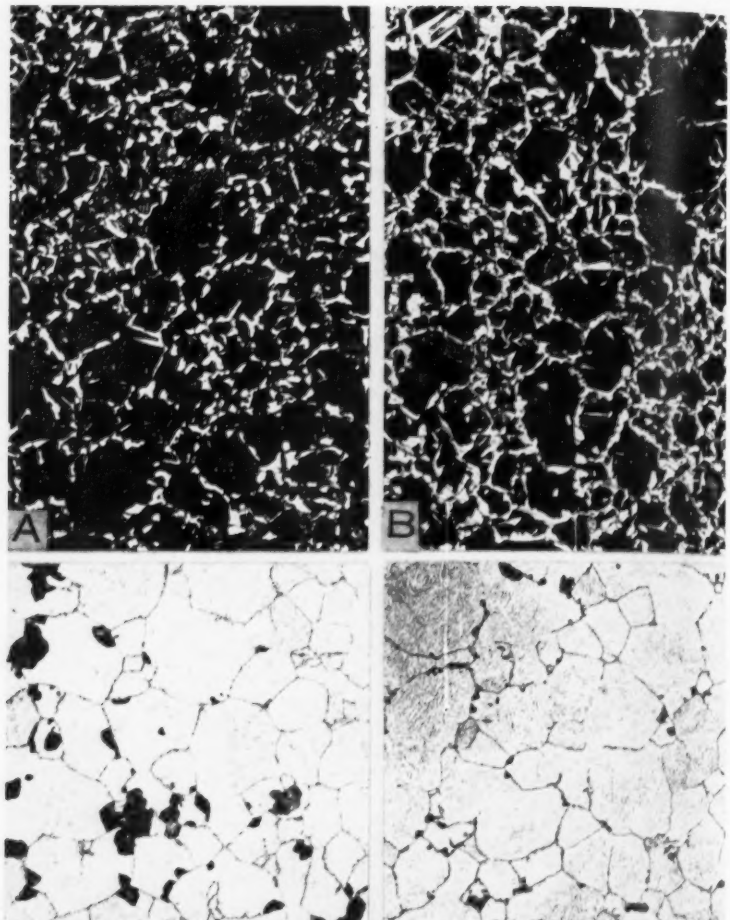

FIGURE 15.—*Grain size at 1,600° F of experimental plain carbon steels (gas carburized).*

A, *GC–1* (0.57 percent of carbon); heated in lead bath; estimated 60 percent of ASTM grain No. 2 to 4, 40 percent of No. 7 to 8. *B*, *GC–1* (0.57 percent of carbon); heated 11 degrees F per minute; estimated 50 percent of ASTM grain No. 2 to 4, 50 percent of No. 7 to 8. *C*, *GC–2* (1.02 percent of carbon); heated in lead bath; 14 grains per square inch at ×100. *D*, *GC–2* (1.02 percent of carbon); heated 7 degrees F per minute; 11 grains per square inch at ×100. Etched with 1-percent nital. ×100.

VI. SUMMARY

The austenitic grain size established at different temperatures with various rates of heating through the transformation range was determined for high-purity alloys of iron and carbon, and for commercial and experimental steels which had a wide range in carbon content. The commercial steels also differed in the method of deoxidation.

For the high-purity alloys of iron and carbon containing 0.50 percent of carbon, the grain size at 1,475°, 1,500°, and 1600° F increased with decrease in rate of heating, whereas the grain size at 1,800° F was not so noticeably dependent upon the rate of heating. Coarser grains were produced at the lower temperatures with slow rates of heating than at higher temperatures with fast rates of heating. With slow rates of heating, grains of the same order of magnitude (coarse grains) were obtained at all temperatures.

With the high-purity alloy of iron and carbon containing 1.01 percent of carbon, the grain size at 1,600° F increased but slightly with decrease in rate of heating. There were indications that for this alloy there is a critical rate of heating at which rate the finest grains were obtained.

With the high-purity alloy of iron and carbon containing 1.21 percent of carbon, the rate of heating had no significant effect on the grain size at 1,800° F.

The rate of heating influenced the grain size at 1,475° F of the commercial 0.49-percent-carbon steels with both noncontrolled and controlled grain size, and at 1,600° and 1,800° F of the noncontrolled steel. The grain size of these steels tended to increase with increase in rate of heating.

With the noncontrolled steel containing 1.05 percent of carbon, the rate of heating had no significant effect on the grain size at 1,650° and 1,800° F. With the controlled steel of similar carbon content, the grain size at 1,650° and 1,800° F was influenced by the rate of heating. At the lower temperature, the coarsest grains were produced with an intermediate rate of heating, whereas at the higher temperature the coarsest grains were produced with the slowest rate of heating.

With the experimental steels, containing approximately 0.5 and 1 percent of carbon, the grain size at 1,600° F was not affected by the rate of heating.

The data presented in this report show that, in certain cases, the austenitic grain size of high-purity alloys of iron and carbon and plain carbon steels is affected by the rate of heating through the transformation range; that is, the grain size of some of these materials is sensitive to the rate of heating. All the materials, however, did not respond in the same manner. In view of these findings it is necessary, when making grain-size specifications, that consideration be given to the possible effects that the rate of heating through the transformation range may have upon the grain size of the steels under consideration.

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METHODS, APPARATUS, AND PROCEDURES FOR THE COMPARISON OF PRECISION STANDARD RESISTORS

By Frank Wenner

ABSTRACT

Reference is made to some of the more important contributions that have been made to the subject of precise measurements of electrical resistance. The sensitivity of bridges when used with the modern high-sensitivity moving-coil galvanometer is discussed rather fully. Special consideration is given to the methods and apparatus used and to the procedures followed in the National Bureau of Standards in those comparisons in which the precision desired is of the order of 1 part in a million.

The more important factors limiting the precision of the comparisons, such as load coefficients, terminals, and contacts, thermoelectromotive forces, insulation, and the optical system of the galvanometer, are discussed rather fully. A method of analysis of networks containing both linear and nonlinear four-terminal conductors is given, and the theoretical basis for the experimental procedure used in determining the effect of slight defects in the insulation is pointed out. This is followed by a brief discussion of Ohm's law from the standpoint of precise resistance measurements and by a brief discussion of units of resistance. Finally, reference is made to more than 100 publications having a more or less direct bearing on the subject of resistance comparisons.

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1. INTRODUCTION

Over a period of many years information pertinent to the comparisons of the resistances of precision standard resistors has been accumulating in this Bureau. This paper presents in a connected form the more important parts of this information not adequately explained in previous publications.

The paper is intended to present:

1. For those having a general interest in electrical measurements, a somewhat comprehensive discussion of the Wheatstone bridge method and of the Thomson bridge method, and a brief description of the apparatus used and procedures followed in this Bureau in making resistance comparisons to a precision of 1 part in 1 million.

2. For those making precise resistance comparisons, a discussion of various factors having a bearing on the precision of such comparisons.

To serve this twofold purpose the work of others previous to 1910 is reviewed briefly, and in this review it is pointed out that the conclusions reached by Heaviside, Maxwell, and others were based on assumptions which do not conform to present conditions. Fundamentals are considered in the earlier parts of the paper, and these are explained rather fully, while highly technical discussions are, for the most part, placed in appendices. The manner of presentation makes it possible for those whose interest is only general to familiarize themselves with some of the fundamentals of precision resistance comparisons without reading more than approximately the first third of the paper, and for those who may be interested only in some particular phase of the problem, such as thermoelectromotive forces or the analyses of networks, to find readily (by referring to the index at the end of the paper) what is given on the subject.

II. REVIEW OF EARLIER WORK

The first measurements, by a null method, of what later was recognized as a definite property of a conductor, called the resistance, were made by Becquerel [3].[1] Using a differential galvanometer, he obtained the first definite proof of the relation between the resistance, length, and cross section of wires. His results were published in 1826. It was in the same year that Ohm [66] published his most important paper, though it has hitherto been generally considered that Ohm's law is of a somewhat later origin. In 1833 Christie [12], who was not familiar with the work of Ohm, described an arrangement of wires which later became known as the Wheatstone bridge. With this connection he verified the results obtained by Becquerel and also determined the relative conductivities of a number of metals. As the significance of Ohm's work was not understood until later, Christie was in the position of having devised one of the best methods of measuring resistance before the concept of resistance had become definitely established.

In 1843 Wheatstone [117, 118] presented a paper before the Royal Society of London, in which he defined resistance and referred to "standard of resistance" and "resistance coils."

In 1862 Thomson [94] published a paper in which he described what he called a "New Electrodynamic Balance for the resistances of short bars and wires." This later became known as the Thomson bridge, the Thomson double bridge, the Kelvin bridge, and the Kelvin double bridge. Thomson seems to have been the first to attempt measurements of the highest precision attainable with the apparatus then available and the first to have even an approximate understanding of the factors limiting the precision of measurement.

Since then the sensitivity of bridges has been a subject of much discussion. Of the more important of the earlier contributions, mention should be made of a paper by Schwendler [79] published in 1866, of a paper by Heaviside [32] published in 1873, a paper by T. Gray [26] published in 1881, and the second and third editions [58] of Maxwell's Electricity and Magnetism, 1892. The conclusions given in these publications were based on an assumption that the battery used was not capable of supplying all of the power desired. Obviously, this assumption would seldom be valid at the present time. Furthermore, no account was taken of the electromotive force developed by the relative motion of the winding and magnet of the galvanometer. This constitutes another reason why the conclusions given are not applicable in case a modern high-sensitivity moving-coil galvanometer is used. However, not all of the earlier writers on this subject were of the opinion that the then available sources of electric power constituted a limiting factor in the attainable sensitivity, since as early as 1862 Thomson, in the paper to which reference has been made, said "I shall conclude by remarking that the sensibility of the method which has been explained, as well as of Wheatstone's balance, is limited solely by the heating effect of the current used for testing." In 1889 Paalzow and Rubens [68], in connection with a study of bolometers, made a rather thorough investigation of the effects of heating by the current in a Wheatstone bridge, but their conclusions are not

[1] Figures in brackets indicate literature references at the end of this paper.

directly applicable to resistance measurements. In 1892 Guye [28], in a further study of bolometers, pointed out that the effect of heating by the test current is proportional to the temperature coefficient of the material from which a bridge arm is constructed, proportional to the square of the current in it, and inversely proportional to its facilities for dissipating heat. He also pointed out that by making all, or pairs, of the bridge arms alike in all respects, the effects of heating by the test current could be almost completely compensated, unless the power dissipation in the bridge arms were unusually large.

In 1893 Glazebrook [23] gave the results of a series of measurements of standard resistors, using different test currents. He found the resistance of 10-ohm standard resistors increased perceptibly as the test current was increased from 0.05 to 0.15 ampere, but he concluded that since the increase in resistance was proportional to the square of the test current, the effect of the smaller test current was extremely small.

In 1895 this was again pointed out by Schuster [78], who evidently was not familiar with the work of Thomson, though he was familiar with that of Guye. Schuster's conclusion was that "The highest percentage accuracy with which a given resistance can be measured is directly proportional to the square root of the maximum electric work which can be done on it without overheating." In 1906 Jaeger [40] and Smith [82, 83, 84], independently, and more recently Von Steinwehr [86], discussed the subject from the same point of view. Nevertheless, the conclusions of Heaviside, Gray, and Maxwell still persist.

The effect of the electromotive force developed by the relative motion of the winding and magnet of the galvanometer is less obvious. However, it has been taken into consideration by Jaeger [40] and by Von Steinwehr [86], both of whom give a different formula for the sensitivity of the Wheatstone bridge (also the Thomson bridge), according to the use of a moving-magnet galvanometer or a suitably damped moving-coil galvanometer.

III. SENSITIVITY OF BRIDGES

1. DAMPING OF GALVANOMETERS

In the modern high-sensitivity moving-coil galvanometer the electromotive force generated in the galvanometer during the time the deflection is changing at its maximum rate may be of the same order of magnitude as the impressed electromotive force. Therefore, during this time it has a marked effect on the current and consequently on the motion of the coil. What is observed is a damping of the motion of the coil, and, among other factors, this depends upon the resistance of the complete galvanometer circuit, that is, the resistance to an electromotive force in the galvanometer branch of the bridge. If this resistance is much less than that which results in critical damping, the movement of the coil toward any new equilibrium position is very sluggish. If, on the other hand, this resistance is considerably more than that which results in critical damping, the coil continues for some time to oscillate about any new equilibrium position. Neither condition is conducive to rapid nor accurate measurements. To obtain a satisfactory performance, either a magnetic shunt or an auxiliary resistance is used for adjusting the damping. As the latter

is more convenient and is more generally used, it only will be considered in deriving expressions for the sensitivity. However, it should be pointed out that in general somewhat higher sensitivities may be obtained by the use of an adjustable shunt on the magnet of the galvanometer.

It is convenient to think of the resistance to an electromotive force in the galvanometer branch as consisting of two parts, namely, the resistance of the galvanometer and the resistance external to the galvanometer. In case the resistance of the bridge between its galvanometer terminals, with the galvanometer branch open, is less than the external resistance which gives a desired damping the auxiliary resistance, U, is connected in series with the galvanometer, as shown in figure 1, and adjusted so as to give the desired damping. In case the resistance of the bridge between the galvanometer terminals is so high as to give insufficient damping, the auxiliary resistance is placed in parallel with the bridge and galvanometer, and so adjusted as to give the desired damping. Some prefer to have the damping critical in all cases. However, measurements can be made somewhat more quickly when the damping is approximately two-thirds critical, which results in an "overshoot" of 6 percent. Also in some cases the sensitivity is higher with the damping less than critical, while in others it is higher with critical damping. Consequently, it is not desirable to use the same damping in all cases. However, to avoid undue complications, it will be assumed, for the present, that the auxiliary resistance will be so adjusted as to give the same damping in all cases, and this will be referred to as the specified damping.

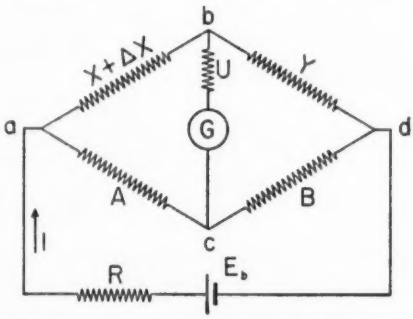

FIGURE 1.—*Circuit diagram of unbalanced Wheatstone bridge.*

Here X, Y, A, and B represent values of the resistances of the four arms for which the bridge would be balanced.

2. WHEATSTONE BRIDGE

In what follows, the arms of the Wheatstone bridge will be designated X, Y, A, and B. When the bridge is balanced, the resistances of the arms will also be designated X, Y, A, and B. To indicate that a reduction of the resistance of the X arm by an amount ΔX would establish a balance of the bridge, the resistances of all arms of the bridge will be designated as shown in figure 1.

Therefore,

$$X/Y=A/B. \tag{1}$$

With the galvanometer branch open, let

E = the potential drop from a to d,
E_x = the potential drop from a to b,
E_a = the potential drop from a to c, and
E_g = the difference between the potential of b and c.

Obviously then

$$E_x=\frac{E(X+\Delta X)}{X+\Delta X+Y}, \tag{2}$$

$$E_a=\frac{EA}{A+B}, \text{ and} \tag{3}$$

$$E_g=\mp\frac{E(X+\Delta X)}{X+\Delta X+Y}\pm\frac{EA}{A+B}. \tag{4}$$

Since the direction of E_g taken as positive is immaterial, to avoid complications in what follows the $\mp$ sign will be taken as positive, and the $\pm$ sign will be taken as negative. Then, since from eq 1 it follows that

$$\frac{A}{A+B}=\frac{X}{X+Y}, \tag{5}$$

eq 4 takes the form

$$E_g=\frac{EX}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right] \text{ or} \tag{6}$$

$$E_g=\frac{EX}{X+Y}\left[(1+\Delta X/X)\left(1-\frac{\Delta X}{X+Y}+\left(\frac{\Delta X}{X+Y}\right)^2-\text{etc.}\right)-1\right]. \tag{7}$$

If the bridge is nearly balanced, second- and higher-order terms may be neglected, in which case eq 7 reduces to

$$E_g=EY\Delta X/(X+Y)^2. \tag{8}$$

Letting dX represent the proportional decrease in the resistance of the X arm, which would establish a balance of the bridge,

$$dX=\Delta X/X. \tag{9}$$

Consequently,

$$E_g=EXYdX/(X+Y)^2. \tag{10}$$

Since the potential difference which would appear across a break were a branch of a network opened, may be considered as an electromotive force acting in that branch, E_g may be considered as an electromotive force in the galvanometer circuit.

Now let D be the change (in scale divisions) of the deflection of the galvanometer resulting from unit change of the electromotive force in the galvanometer circuit when the resistance V connected in series with the galvanometer is that which gives the specified damping. Hereafter D will be referred to as the sensitivity of the galvanometer. Also let W be the resistance of the bridge between its galvanometer terminals, b and c of figure 1, with the galvanometer branch open. The case in which W is less than V will be considered first. Then U is placed in series with the galvanometer, as shown in figure 1, and so adjusted that

$$U+W=V.$$

Consider that the zero of the galvanometer scale is at an end of the scale, that Q is the scale reading with the battery branch of the bridge open, that Q_1 is the scale reading with the battery connected

as shown in figure 1, that Q_2 is the scale reading with the leads to the battery interchanged, and that the galvanometer is so connected that, with dX positive, $Q_1 > Q > Q_2$. Then from eq 10 it follows that

$$Q_1 - Q = DEXYdX/(X+Y)^2, \tag{11}$$

and

$$Q_2 - Q = -DEXYdX/(X+Y)^2. \tag{12}$$

Subtracting eq 12 from eq 11 eliminates Q and gives

$$dQ = 2DEXYdX/(X+Y)^2, \tag{13}$$

where $dQ(=Q_1 - Q_2)$ is the change in the deflection of the galvanometer following a reversal of the connections to the battery.

Now let S represent the combined sensitivity of the bridge and galvanometer, that is, define S by the equation

$$S = dQ/dX. \tag{14}$$

Then it follows from eq 13 and 14 that

$$S = 2DEXY/(X+Y)^2. \tag{15}$$

Since from eq 1 it follows that

$$XY/(X+Y)^2 = AB/(A+B)^2, \tag{16}$$

another expression for the sensitivity is

$$S = 2\ DEAB/(A+B)^2. \tag{17}$$

However, if the resistance W of the bridge between its galvanometer terminals is greater than the resistance V, which gives the specified damping of the galvanometer, the resistance U is placed in parallel with the galvanometer and adjusted so that

$$UW/(U+W) = V. \tag{18}$$

From eq 10 it follows that with breaks in both the U and galvanometer branches the potential drop across each of the breaks

$$E_{gu} = EXYdX/(X+Y)^2. \tag{19}$$

With the galvanometer branch only open, the current in the U branch is $E_{gu}/(U+W)$, while the potential drop across the break in the galvanometer branch, E_g, is U times this current, or $E_{gu}U/(U+W)$. Therefore, since $U/(U+W) = V/W$.

$$E_g = EXYdXV/W(X+Y)^2. \tag{19a}$$

That is, the effect of the resistance in parallel with the galvanometer (in parallel with the bridge, if considered from the standpoint of the electromotive force developed in the galvanometer coil as a result of its motion), when of such value as to give the specified damping of the galvanometer, is a reduction of the electromotive force in the galvanometer circuit, and consequently of the combined sensitivity

of the bridge and galvanometer by the ratio of V to W [101]. Therefore, instead of the relations given by eq 15 and 17,

$$S=2DE\ \frac{XYV}{(X+Y)^2W}, \tag{20a}$$

and

$$S=2DE\ \frac{ABV}{(A+B)^2W}. \tag{20b}$$

With reference to these equations it should be noted: (1) That although they are first-order approximations, for the purpose at hand they may be considered as exact, and it is immaterial whether the potential drop across the bridge is measured with the galvanometer branch (and its parallel branch) open or closed. (2) That if they are to apply with the resistance used in adjusting the damping of the galvanometer either in series or in parallel with the galvanometer, in all cases in which the ratio of V to W is greater than one it is to be taken as one.

In what follows, the relative positions of the battery and galvanometer shown in figure 1 (and fig. 2) will be considered as their normal positions. With the position of the galvanometer and battery and their respective rheostats interchanged,

$$S=2D\epsilon\ \frac{AXV}{(A+X)^2W}, \tag{21a}$$

and

$$S=2D\epsilon\ \frac{BYV}{(B+Y)^2W}. \tag{21b}$$

Here ϵ is the potential difference between branch points b and c, which now are the battery terminals of the bridge.

Except for limitations imposed by the heat developed in the bridge arms by the test current, eq 20 and 21 are in convenient form for use. However, if E (or ϵ) exceeds a certain magnitude, the heating in the bridge will result in a change of the resistance of one or more of the bridge arms by an amount in excess of that permissible or in excess of that corresponding to the precision sought in the measurement. There is, therefore, a fairly definite upper limit to the sensitivity which may be used in any particular case. This will be referred to as the permissible sensitivity. The permissible sensitivity depends on the sensitivity of the galvanometer to an electromotive force in a circuit giving the specified damping, the resistance external to the galvanometer which gives the specified damping, the relative magnitudes of the bridge arms, the resistance of the bridge between its galvanometer terminals (factors which have been considered above), the precision sought in the measurement, and the load coefficients of the bridge arms.

The load coefficient of a conductor will be defined as the ratio of the proportional change in its resistance to the power dissipated in it. It would be logical therefore to develop formulas for the sensitivity of bridges based explicitly on the power dissipated in each bridge arm. However, there is some advantage in using either the current [40] or

the potential drop in each bridge arm, and here the potential drop will be used. How these factors, especially the precision desired in the measurement and the load coefficients, limit the permissible sensitivity may be seen by considering the following example.

Assume that

$D=10$ millimeters per microvolt,
$V=15$ ohms,
$X=10$ ohms,
$Y=5$ ohms,
$A=50$ ohms, and
$B=25$ ohms.

Assume that the bridge is to be balanced to 1 part in a million and that investigation of the load coefficients has shown that a change of 1 part in a million occurs

in X when E_x, the potential drop in X, is 0.75 volt,
in Y when E_y, the potential drop in Y, is 0.75 volt,
in A when E_a, the potential drop in A, is 2 volts, and
in B when E_b, the potential drop in B, is 1.5 volts.

Obviously, if the resistance of X is to be calculated from values assigned to A, B, and Y, the potential drop in none of the four resistances can be permitted to exceed the value just stated, and preferably it should not exceed two-thirds this value. In the absence of known compensating effects, it will be assumed that the maximum permissible sensitivity is obtained when the potential drop in the bridge is as high as possible, without that in any arm of the bridge being higher than two-thirds that which results in a proportional change in the resistance equal to the precision sought in the measurement.

With the battery and galvanometer in their normal positions, it is readily seen that $W=20$ ohms, also that as E is increased E_x is the first to reach the maximum permissible value and that $E_x=EX/(X+Y)$. Therefore, eq 20*a* may be written

$$S=2DE_x\frac{YV}{(X+Y)W}, \tag{22}$$

and taking $E_x=0.5$ volt gives

$$S=\frac{2\times10\times10^6\times.5\times5\times15}{15\times20}=2.5\times10^6. \tag{22a}$$

That is, the change in deflection of the galvanometer following a reversal of the connections to the battery is 2.5 mm per part per million lack of balance of the bridge.

With the positions of the battery and galvanometer interchanged, it is readily seen that W is less than V; that, as ϵ is increased, E_b is the first to reach the maximum permissible value; and that $E_b=\epsilon B/(B+Y)$. Therefore, eq 21*b* may be written

$$S=2DE_b\frac{YV}{(B+Y)W}. \tag{23}$$

Taking $E_b=1$ volt and $V/W=1$ gives

$$S=\frac{2\times10\times10^6\times1\times5\times1}{30}=3.3\times10^6, \tag{23a}$$

or a deflection of 3.3 mm per part per million lack of balance of the bridge.

It will thus be seen that either arrangement of battery and galvanometer gives a permissible sensitivity more than ample for establishing balances of the bridge to 1 part per million, and that there is not much choice between the two.

If the bridge were to be balanced to 1 part in 4 million, the permissible potential drop in each bridge arm, and consequently the permissible sensitivity, would be only half as large, while the precision sought is higher by a factor of 4. Consequently, the permissible residual deflection of the galvanometer could be only one-eighth of what it might equally well be in making a balance to 1 part in a million. On the other hand, if the load coefficients were smaller by a factor of 4, the permissible potential drops would be higher by a factor of 2, and consequently the permissible sensitivity would be higher by a factor of 2.

Returning now to a general consideration of the sensitivity of the Wheatstone bridge, it should be noted that with the battery and galvanometer in their normal positions,

$$E=E_x(X+Y)/X=E_y(X+Y)/Y=E_a(A+B)/A=E_b(A+B)/B, \quad (24)$$

and with the positions of the battery and galvanometer interchanged,

$$e=E_x(A+X)/X=E_y(B+Y)/Y=E_a(A+X)/A=E_b(B+Y)/B, \quad (25)$$

Therefore, it follows from eq 20 and 24 that

$$S=2DE_x\frac{YV}{(X+Y)W}, \quad (26x)$$

$$S=2DE_y\frac{XV}{(X+Y)W}, \quad (26y)$$

$$S=2DE_a\frac{BV}{(A+B)W}, \quad (26a)$$

and

$$S=2DE_b\frac{AV}{(A+B)W}, \quad (26b)$$

and from eq 21 and 25 that

$$S=2DE_x\frac{AV}{(A+X)W}, \quad (27x)$$

$$S=2DE_y\frac{BV}{(B+Y)W}, \quad (27y)$$

$$S=2DE_a\frac{XV}{(A+X)W}, \quad (27a)$$

and

$$S=2DE_b\frac{YV}{(B+Y)W}. \quad (27b)$$

Now, if it is understood that E_x, E_y, E_a, and E_b each represents the maximum permissible potential drop in X, in Y, in A, and in B, each of eq 26 and each of eq 27, in general, gives a different sensitivity. However, that one of eq 26 which gives the lowest sensitivity gives the maximum permissible sensitivity with the battery and galvanometer in their normal positions; while that one of eq 27 which gives

the lowest sensitivity gives the maximum permissible sensitivity with the positions of the battery and galvanometer interchanged. This statement concerning the maximum permissible sensitivity is based on the assumption that the resistance of one arm of the bridge is to be calculated from known values of the resistances of the other arms. Later it will be shown that measurements may be made in such a way as to largely eliminate the effect of heating by the test current in one or more of the bridge arms, and that the effect of heating in all bridge arms, if not excessive, may be determined experimentally and an appropriate correction applied. In either case, a somewhat higher sensitivity may be permissible.

It will be noted that by keeping the galvanometer branch closed and reversing the connections to the battery, the permissible sensitivity is twice that which would be obtained by first closing the battery branch and then the galvanometer branch. In addition this eliminates the effects of thermoelectromotive forces and leakage from power circuits, insofar as these remain constant over a time corresponding to a few periods of the galvanometer. Furthermore, a few reversals of connections to the battery, at intervals corresponding approximately to the period of the galvanometer, with the galvanometer branch closed eliminate the effect of gradual drifts and hysteresis in the galvanometer deflections. This is of great importance, since usually in measurements of the highest precision, adjustments must be carried to a point at which the changes in the deflection of the galvanometer resulting from a lack of perfect balance of the bridge are much less than the changes in the deflection resulting from disturbing influences.

3. THOMSON BRIDGE

Referring to figure 2, a material simplification in the analysis may be brought about by using Kennelly's [48] Δ to Y transformation, that

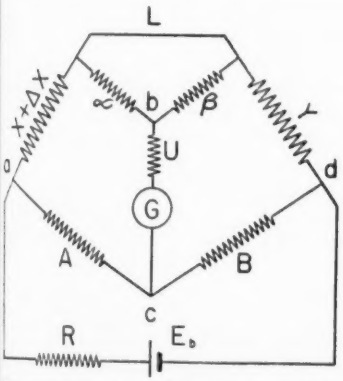

FIGURE 2.—*Circuit diagram of unbalanced Thomson bridge.*

Here X, Y, A, B, α, and β represent resistances of the six arms for which the bridge would be balanced.

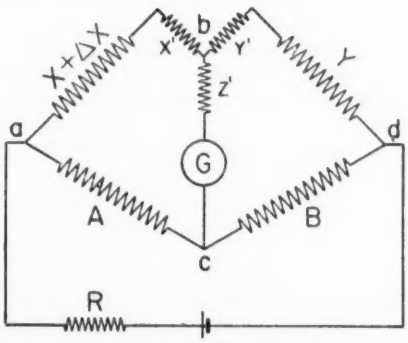

FIGURE 3.—*Circuit diagram shown in figure 2, with the delta connected resistances, α, β, and L, replaced by the equivalent star-connected resistances, X', Y', and Z'.*

is, by considering the three Δ connected conductors, L, α, and β, as replaced by three Y connected conductors, X', Y', and Z', as is shown in figure 3.

This is the same as the Wheatstone bridge, except that there is a resistance X' in series with X, Y' in series with Y, and Z' in series with the galvanometer.

The procedure followed in deriving eq 26 and 27 then leads to

$$S=2DE_x\frac{(Y+Y')V}{(X+X'+Y+Y')W}, \tag{28x}$$

$$S=2DE_y\frac{(X+X')V}{(X+X'+Y+Y')W}, \tag{28y}$$

$$S=2DE_a\frac{BV}{(A+B)W}, \tag{28a}$$

and

$$S=2DE_b\frac{AV}{(A+B)W}, \tag{28b}$$

if the positions of the battery and galvanometers are as shown in figure 2, and

$$S=2DE_x\frac{AV}{(A+X+X')W}, \tag{29x}$$

$$S=2DE_y\frac{BV}{(B+Y+Y')W}, \tag{29y}$$

$$S=2DE_a\frac{(X+X')V}{(A+X+X')W}, \tag{29a}$$

and

$$S=2DE_b\frac{(Y+Y')V}{(B+Y+Y')W}, \tag{29b}$$

if the relative positions of the battery and galvanometer are interchanged.

With reference to the differences between eq 26 and 28 and eq 27 and 29, it should be pointed out that for the Y-connected conductors to be equivalent to the Δ-connected conductors it is necessary that

$$\begin{aligned} X'&=L\alpha/(L+\alpha+\beta),\\ Y'&=L\beta/(L+\alpha+\beta), \qquad (30)\\ \text{and}\\ Z'&=\alpha\beta/(L+\alpha+\beta). \end{aligned}$$

In all cases, X' and Y' are each less than L, and usually L is very small in comparison with $A+X$ and $B+Y$. Consequently, there are relatively few cases in which X' and Y' may not be omitted from eq 29x and 29y, which then become identical with eq 27x and 27y. In few if any cases is the permissible sensitivity determined by the power dissipated in A or B. Therefore there is no need for drawing conclusions from either eq 29a or eq 29b. The Thomson bridge method, with the positions of the battery and galvanometer interchanged, is not used in the comparison of precision standard resistors. However, the Thomson bridge method may be used in resistance

thermometry, and then there are advantages in interchanging the positions of the galvanometer and battery.

With reference to eq 28*x* and 28*y*, which apply when the battery and galvanometer are in their normal positions (see figs. 2 and 9), as a balance is approached by the procedure to be described later, $(Y+Y')/(X+X'+Y+Y')$ approaches $Y/(X+Y)$ and $(X+X')/(X+X'Y+Y')$ approaches $X/(X+Y)$. Equations 28 may, therefore, be considered to be the same as eq 26. However, with the battery and galvanometer in their normal positions and with A, B, X, and Y the same in both the Thomson bridge and the Wheatstone bridge, W is higher in the Thomson bridge than in the Wheatstone bridge. In the comparisons of precision standard resistors made in this Bureau by the Thomson bridge method, this is of no consequence, since W is less than V. Under these conditions, the sensitivity of the Thomson bridge is the same as that of the Wheatstone bridge.

It is improbable that a case might arise in which the power dissipated in the α arm or the β arm of the bridge would limit the permissible sensitivity, whether the positions of the battery and galvanometer are normal or interchanged. Therefore, no equation containing the potential drop in α or in β is given.

IV. LOAD COEFFICIENTS

Reference has already been made to load coefficients defined as the ratio of the proportional increase in the resistance to the power dissipated. This definition requires some amplification, since, in all cases, time and the medium surrounding the resistor are involved. In the following discussion it will be assumed that the resistors are immersed in oil of low viscosity and that this oil is kept in fairly rapid circulation. Fortunately, most precision resistors come to an approximate temperature equilibrium with the oil in a fairly short time, usually less than 1 minute. With resistors of the type developed in this Bureau about 30 years ago [76], which are sealed in cases containing oil, the change in resistance is rapid at the start of the measuring current and later is very gradual. These resistors apparently have two thermal-time constants, one of about 30 seconds and one of about 30 minutes. Stated in another way, the difference in temperature between the resistance element and the oil in the sealed container becomes nearly constant in a minute, while the difference in temperature between the oil in the container and the oil of the bath becomes nearly constant in an hour. The load coefficients stated for standards of this type apply when the current has been passing from 1 to 2 minutes, the time usually required for making a measurement.

From what has just been said it might be assumed that load coefficients are proportional to temperature coefficients and inversely proportional to the facilities provided for dissipating heat. Experience shows that, in general, this is so only if the design is such that the heating by the test current results in no marked mechanical strain. Consequently, in measuring load coefficients it may be better, when possible, to use a procedure such that this assumption is reduced from a first to a second or third order of importance.

As an illustration of this point consider that it is desired to determine the load coefficient of a 1-ohm standard. In that case the procedure might be as follows:

(1.) From among the available 0.1- and 0.01-ohm standards select from each denomination one of the better from the standpoint of facilities for dissipating heat and low temperature coefficient.

(2.) Inspect these two standards and make an estimate of their relative facilities for dissipating heat.

(3.) From the estimate of their relative facilities for dissipating heat and their known temperature coefficients make an estimate of the ratio of the load coefficient of the 0.01-ohm standard to the load coefficient of the 0.1-ohm standard, and designate this ratio k.

(4) Place the 0.1-ohm standard in the X arm and the 0.01-ohm standard in the Y arm of a bridge, and balance the bridge by adjustments of the A arm, first with 0.1-watt and then with 0.5-watt power dissipation in the 0.1 ohm-standard. Assuming that the power dissipated in the A and B arms of the bridge has no appreciable effect on their resistances, the load coefficient of the 0.1-ohm standard (that is, the change in the resistance of the 0.1-ohm standard resulting from the dissipation of 1 watt in it) is taken as 2½ times the proportional increase in the resistance of the A arm divided by $(1-0.1\ k)$.

(5) Place this 0.1-ohm standard in the Y arm of the bridge and a 1-ohm standard in the X arm, and balance the bridge first with one and then another potential drop across the bridge. The load coefficient of the 1-ohm standard is taken as equal to the proportional increase in the resistance of the A arm of the bridge divided by the increase in power dissipation in the 1-ohm standard plus 0.1, the load coefficient of the 0.1-ohm standard as determined in (4). It will be noted that the result obtained involves the initial estimate of the load coefficients of the 0.01-ohm standard relative to that of the 0.1-ohm standard to the extent of only 1 percent. The value thus obtained for the load coefficient of the 1-ohm standards may, therefore, be presumed to be somewhat more precise than the value obtained for the load coefficient of the 0.1-ohm standard.

Load coefficients of standards of higher nominal values may be determined as outlined above, but for standards of the lowest nominal value used a different procedure is required. If among these there are two standards of such construction that it may be assumed that the temperature rises for equal power dissipations are equal and that the temperature inequalities cause little or no mechanical strain, and if these two standards have markedly different resistance-temperature coefficients, their load coefficients may be determined from their resistance-temperature coefficients and the difference of their load coefficients obtained by direct comparison.

Having determined the load coefficients of one or more standard resistors, the load coefficients of others of the same nominal value are readily determined by direct comparisons, whether or not their performances are normal.

Investigation of wire standard resistors of the Physikalisch-Technische Reichsanstalt design having winding areas of approximately 40 cm^2 has shown that the proportional change in their resistance resulting from the dissipation of 1 watt in their windings is about the same as the proportional change in their resistance resulting from increasing the temperature of the oil bath 1° C. It may be concluded therefore that the temperature rise of the resistance material above the temperature of the oil is about 1° C per watt power dissipation.

On the same basis it has been concluded that for the sealed standard resistors developed in this Bureau about 1907 and the double-walled type constructed prior to 1930 [89] the temperature rise is also about 1° C per watt power dissipation. For standard resistors of lower denominations in which the resistance material is in the form of sheets, the temperature rise per watt dissipation is less. For those of the Physikalisch-Technische Reichsanstalt design, of the smaller size, the temperature rise is about 0.2° C for the 0.01 ohm, 0.4° C for the 0.001 ohm, and 0.3° C for the 0.0001 ohm per watt dissipation.

Since most of the temperature coefficients are less than 20 parts per million, errors resulting from heating will, in general, be less than 1 part in 2 million if the potential drops are limited to the following values:

0.15 volt for 1-ohm,
.5 volt for 10-ohm,
1.5 volts for 100-ohm,
5. volts for 1,000-ohm,
15. volts for 10,000-ohm,
0.05 volt for 0.1-ohm,
.035 volt for 0.01-ohm,
.0075 volt for 0.001-ohm, and
.0025 volt for 0.0001-ohm standard resistors.

The temperature rises, and consequently the load coefficients, depend on the viscosity and other factors affecting the circulation of the oil, the design of the standards, and their temperature coefficients, so there must of necessity be large variations. However, the potential drops stated above are about the maxima permissible in measurements to 1 part per million, unless the load coefficients are known to be abnormally low, or a procedure is followed for eliminating the error which otherwise would result from the heating by the test current.

In routine testing the schedule of potential drops given above is followed approximately in the comparisons of resistors having nominal values of 1 ohm and less. In the comparisons of resistors having nominal values of 10 ohms and more the potential drops used are much less than those given in the schedule. In special cases the criterion used for the maximum permissible potential drops, without an application of a correction for the heating, is a very small but definitely noticeable change in the balance of the bridge on increasing the potential drop across the bridge by a factor of 2.

If there is occasion to use larger potential drops, if there is reason to suspect one or more abnormally large load coefficients, or if the load coefficient of one or more of the bridge arms is not known to the accuracy necessary for obtaining the desired precision, a valid correction for the effect of the heating may be obtained by balancing the bridge first with what is presumed to be a suitable potential drop across the bridge and second with a somewhat larger potential drop across the bridge. If the second potential drop is twice the first, the effects of the temperature rise in each arm of the bridge will be 4 times as large as with the first potential drop. Consequently, if the balances are established by adjustments of the A arm (or A and α arms) of the bridge and the value taken for A is four-thirds the first minus one-third the second, a correction is applied for the effects of heating by the test current in all arms of the bridge.

V. GALVANOMETER USED WITH NBS PRECISION BRIDGE

The galvanometer which has been and still is being used in most of the precision resistance measurements made in the National Bureau of Standards was designed and constructed especially for the purpose about 1914 [116]. It has an adjustable shunt on the magnet, all-copper circuit, and taut suspensions with the center of gravity of the moving system slightly off the axis of rotation. This latter feature makes it possible by tilting to adjust the period, T, over a range from about 5 to about 15 seconds. In cases in which really high sensitivity is desired the performance seems to be most satisfactory with a period of about 10 seconds. The shunt of the magnet is so adjusted that with a period of 10 seconds the external resistance giving critical damping is 35 ohms. This adjustment once made is seldom changed as the shunt is not readily accessible. The galvanometer is used with a scale distance of 1.5 m. With critical damping the operating constants then are

$$T=10 \text{ seconds},$$
$$D=30 \text{ mm}/\mu\text{v},$$
$$V=35 \text{ ohms},$$

or with two-thirds critical damping, which results in a 6 percent overshoot and gives about the maximum speed of operation with the 10 second period, the operating constants are

$$T=10 \text{ seconds},$$
$$D=20 \text{ mm}/\mu\text{v},$$
$$V=60 \text{ ohms}.$$

It may be of interest to see what the permissible sensitivity is in the comparison of 10-ohm standards when the A and B arms of the bridge are each 25 ohms, the battery is in the normal position, and the galvanometer is used critically damped. In this case the resistance, W, of the bridge between galvanometer terminals is 17½ ohms. With E_x or E_y=0.5 volt, it follows from eq 26_x or 26_y that $S=2\times30\times.5\times10/20=15$ mm per part per million lack of balance of the bridge. This is not only much higher than is needed but is higher than it is desirable to use. In some cases, however, the permissible sensitivity is none too high, and in a very few cases it is not quite sufficient for the establishment of balances to 1 part per million. In these the potential drop may be increased to the point at which errors resulting from heating (calculated from a knowledge of the load coefficients of the particular standards used) and from lack of sensitivity are approximately equal, or even beyond this point, and then a correction for the heating is determined and applied. In a very few cases there would be an advantage in using a galvanometer of higher sensitivity. However, for a considerable part of the measurements the galvanometer is adjusted so as to have a period of from 6 to 8 seconds. Then both the sensitivity and the external resistance giving a specified damping are less than with the 10-second period.

VI. NBS PRECISION BRIDGE

The bridge now in use in the comparisons of precision standard resistors was designed and constructed in this Bureau in 1918. All parts of the bridge arms and the adjustable resistor used in regulating

the damping of the galvanometer are immersed in oil. The ammeter, voltmeter, and rheostats used in regulating the test current and other auxiliary equipment are conveniently located outside the oil bath. The entire bridge circuit is shielded against leakage from direct-current power circuits. The oil bath is thermostatically controlled at a temperature of 25° C, and during the time the apparatus is in use the oil is kept in vigorous circulation. During the summer the normal dew point is occasionally very nearly 25° C. To prevent the condensation of moisture in the oil bath and to maintain good insulation of the battery, galvanometer, and other parts of the circuit outside the oil bath, the air of the laboratory is dried by refrigeration.

The more important resistance sections of the bridge are of the double-walled sealed type developed jointly with others [98] of the Bureau's staff. The cases contain no oil, since the use of oil in permanently sealed resistors has long been considered inadvisable. To obtain low load coefficients the cases were made considerably larger than those first described, and the resistance wire was selected on the basis of low temperature coefficient. The primary of a well-insulated variable mutual inductor is connected in series with one of the battery leads, and its secondary is connected in series with one of the galvanometer leads. This inductor (not shown in fig. 5) serves to balance the electromotive force induced in the galvanometer circuit on reversing the test current.[2] The *A*-arm of the bridge is of the adjustable direct-reading type, such as is discussed in a recent publication [63] of this Bureau. It consists of five resistance sections, three of which are adjustable by means of dial switches. These three sections are shown in figure 4. With each of the dial switches set at its mid-position, the current in each shunt is one-tenth of that in the section shunted, and the sum of the resistances of the three

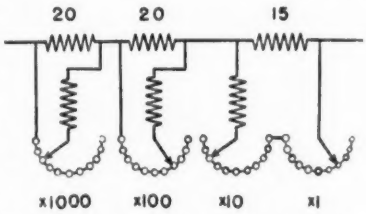

FIGURE 4.—*The part of the A arm of the NBS precision bridge which is adjustable in steps of 0.0001 ohm over the range from approximately 49.4445 ohms to approximately 50.5555 ohms.*

[1] The lack of an inductive balance manifests itself as ballistic deflection of the galvonometer, following a reversal of the test current. If this ballistic deflection is large, it limits, somewhat, the precision of the resistive balance. However, there is a more important reason why the ballistic deflections should be kept small. Most sensitive galvanometers when deflected alternately in one and then in the other direction, by equal amounts, have their rest points shifted slightly in the direction of the last deflection. For the galvanometer used with the NBS precision bridge, if the rest point is observed after a deflection in one direction and again after an equal deflection in the opposite direction, the difference between the two observed rest points is from 1 to 2 percent of the amplitude of the deflections. If, therefore, systematic errors from this source are to be insignificant, inductive balances must be such that the ballistic deflections are less than 50 times the change in deflection resulting from lack of resistive balances corresponding to the precision sought in the comparisons.

In the comparisons of standard resistors of the usual construction, having resistances in the range from 0.1 to 100 ohms, usually this condition is realized without a special device for making inductive balances and without special precautions on the part of the observer. However, if the resistors in the *X* and *Y* arms of the bridge have low resistances of different nominal values (such, for example, as 0.001 and 0.0001 ohm), and if a high precision is desired, a means for compensating the effect of the difference between their time constants and of mutual inductances between different parts of the bridge circuit is necessary. Likewise, if the resistors in the *X* and *Y* arms of the bridge are of the usual bifilar construction and have high resistances of different nominal values (such, for example, as 10,000 and 1,000 ohms), and if a high precision is desired, provision should be made for compensating the effect of the distributed capacitances. The effects of self and mutual inductances and of distributed capacitances may be compensated by a mutual inductor having one of its windings connected in a galvanometer lead and the other winding connected in a battery lead between the current-reversing switch and a current terminal of the bridge, provided the inductance is adjustable over a suitable range of positive and negative values.

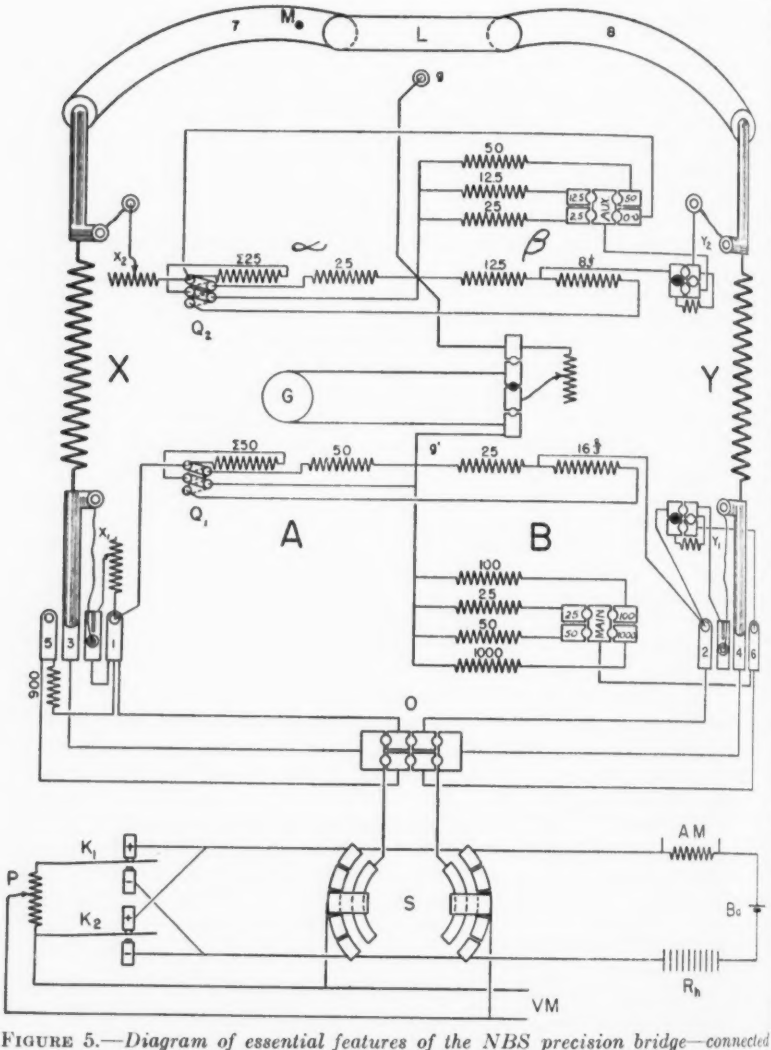

FIGURE 5.—*Diagram of essential features of the NBS precision bridge—connected as a Thomson bridge.*

In general, the notation here is the same as in figures 2, 9, and 16. Here 1 and 2 represent main terminal blocks (shown in detail in fig. 11); 3 and 4, current-terminal blocks; 5 and 6, terminal blocks for the less frequently used ratios of A to B; 7 and 8, hinged terminal blocks to accommodate standard resistors with different spacing of terminals; Σ 50, the three adjustable resistance sections shown in figure 4; R_h, compression carbon-plate rheostat for adjusting large test currents; P, potential divider for adjusting small test currents; AM, ammeter shunt; VM, voltmeter leads; k_1, k_2, and S, keys and switch for closing battery branch and reversing connections to battery leads, k_1 and k_2 for small test currents, S for large test currents; Q_1 and Q_2, copper links and terminal blocks with amalgamated contacts for connecting two sections of the A arm and of the a arm, either in series or in parallel. For the Wheatstone bridge method usually terminals 1 and 2 are used instead of 3 and 4, x_2 and y_2 are left disconnected, and the galvanometer circuit is closed by a connection between M and g.

sections with their shunts is 50 ohms. The resistance sections of the shunts are so chosen that the steps of the dial switches change the combined resistances of sections and shunts respectively by 0.0001, 0.001, 0.01, or by 0.1 ohm. The positions of each dial switch are numbered from 0 to 10. When the three shunted sections designated Σ50 in figure 5, are connected in series, or in parallel, with the 50-ohm section of the A arm, and any section of the B arm of the bridge used, differences in the readings of the dial switches correspond to differences in the ratio of A to B in parts per million of the nominal ratio of A to B. When the three shunted sections are connected in series with the 50-ohm section and the 900-ohm section, differences in the readings of the dial switches correspond to differences in the ratio of A to B in parts in 10 million of the nominal ratio of A to B. In all cases the A arm has its nominal resistance when the reading of the dial switches is approximately 5555.

The dial switches may be rotated indefinitely in either direction, while the complete range of adjustment is covered by a rotation of slightly less than 120°. The brushes, three in number for each switch, are of the multiple-leaf type, such as are used by Otto Wolff. They are mounted on what amounts to the feet of a rigid tripod, and the contact pressure is supplied by three coiled steel springs. This arrangement largely eliminates rocking of the brushes as the switch is rotated and gives a nearly constant pressure of the switches against the contact blocks. With the switch in any position, one of the brushes rests on an insulated segment and the circuit is through the other two brushes in series.

The dial switches are of good quality and operate under oil. Presumably, therefore, variation of the resistance of the contacts of each dial switch never exceeds 0.001 ohm. Since the resistance of each shunt is 10 times the resistance of the section shunted, a variation of 0.001 ohm in each of the four dial switches in the same direction results in a variation of the combined resistance of sections and shunts of 0.004/121 ohm. This is 1 part in 3 million in case A is nominally either 25 or 100 ohms, and 1 part in 30 million in case A is nominally 1,000 ohms. However, the probable effects of the variations of the resistances of the dial switches are approximately one order smaller than this.

The B arm (see fig. 5) has resistance sections of 16⅔, 25, 50, 100, and 1,000 ohms, most of which may be used singly or in combinations. Therefore, a number of values for the ratios of A to B may be had, such as 1 to 1, 2 to 1, 4 to 1, 10 to 1, 1 to 2, 1 to 4, and others seldom used. Each of these ratios may be varied by changing the readings of the dial switches of the A arm, and for the most part the range of variation is from 0.5 percent below to 0.5 percent above the nominal ratio.

Adjustments are such that differences in the readings of the dial switches, not in excess of 500, correspond to differences in the ratio of A to B well within 1 part in a million of the nominal ratio providing no one of the readings differs by more than 500 from that for which A has its nominal value. In exceptional cases corrections must be applied to the readings to give differences accurate to 1 part in a million of the ratio.

The α and β arms (see fig. 5) are similar to the A and B arms, except that the resistance of corresponding sections is half as large, the α arm

does not have a 450-ohm section corresponding to the 900-ohm section of the A arm, and the β arm does not have a 500-ohm section corresponding to the 1,000-ohm section of the B arm. The dial switches of A and α arms are mechanically connected so as to always have the same reading.

A general view of the bridge removed from the oil is shown in figure 6. A general view of the bridge and oil bath is shown in figure 7. Two features of the oil bath which should be mentioned are the means employed for securing a reasonably uniform temperature throughout the bath and of maintaining the desired temperature. The oil bath has a false bottom supported about 3 cm above the true bottom of the bath and extending to within about 1 cm of the side walls, which are of nickel-plated copper. Near the center of the false bottom there is an opening about 8 cm in diameter, and the circulating propeller is located beneath this opening and concentric with it. The circulation of the oil in the central part of the bath is downward, through this opening, outward between the false and true bottoms, upward next to the side walls to near the surface of the oil, and inward on and under the oil surface. The rate of the circulation is approximately 3 liters per second, so that the volume of oil passing through the central opening in 1 minute is approximately the same as the volume of oil contained in the bath. The heat for maintaining the temperature is supplied by small carbon-filament lamps located slightly above and near the opening in the false bottom. By means of radiation, a part of the heat developed in the lamp filaments is distributed through the oil almost instantly. That part of the radiation from the lamp filaments absorbed in and near the lamp bulbs is carried quickly to the side walls of the bath and surface of the oil, the parts from which heat is being lost by radiation and air convection.

VII. BRIDGE EQUATIONS

In the usual discussion of the Wheatstone bridge it is assumed that if X represents the unknown resistance, its magnitude is to be calculated from the equation

$$X=YA/B \tag{32}$$

That is, it is assumed that magnitudes of Y, A, and B are each known to an accuracy at least as high as that expected for X, or that either the magnitude of Y or A and the ratio of A to B or Y to B are known to this accuracy. In measurements of the type under consideration neither of these assumptions can, in general, be made. It is therefore necessary to use some procedure which avoids the use of these assumptions.

1. SUBSTITUTION METHOD

If a number of nominally equal resistances X_1, X_2, X_3, X_4, etc. are to be measured and the resistances of two or more of these are known, for example X_2 and X_4, usually a standard of the same nominal value as those being measured is placed in the Y arm, and the B arm is made nominally equal to the A arm. Then all of the X's may be substituted one after the other in the X arm, and the bridge balanced in each case by an adjustment of the A arm. If a_1, a_2, a_3, a_4, etc., are the read-

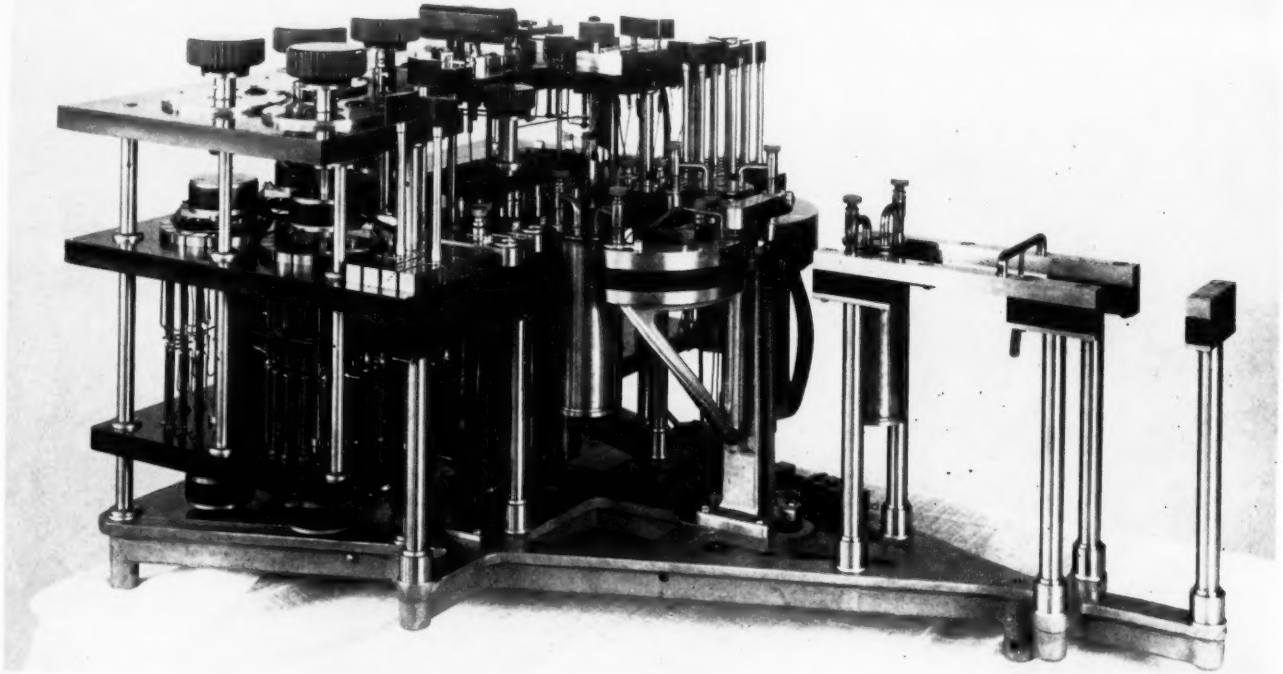

FIGURE 6.—*General view of the NBS precision bridge (removed from oil bath).*

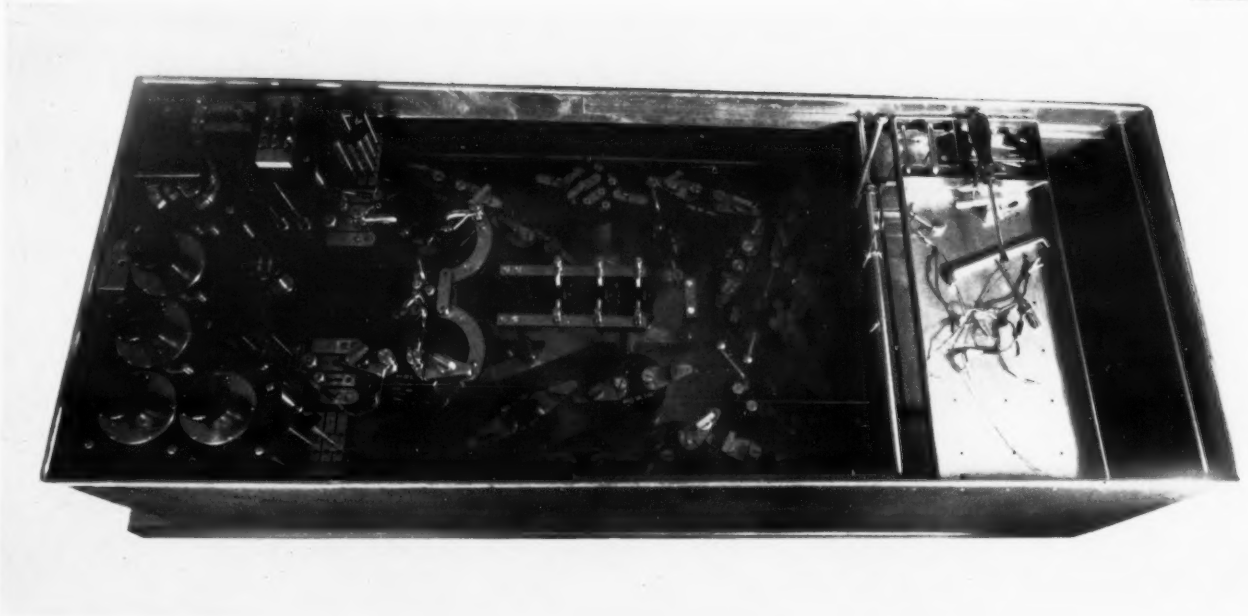

FIGURE 7.—*General view of the NBS precision bridge and oil bath.*

ings of the dial-switch settings, A_n is the nominal value of A, and a_n is the reading of the dial-switch settings for which $A=A_n$,

$$\left.\begin{aligned} X_1&=\frac{Y}{B}A_n(1+a_1-a_n)\\ X_2&=\frac{Y}{B}A_n(1+a_2-a_n)\\ X_3&=\frac{Y}{B}A_n(1+a_3-a_n)\\ X_4&=\frac{Y}{B}A_n(1+a_4-a_n)\end{aligned}\right| \qquad (33)$$

etc.

Letting N represent the nominal value of X and of YA_n/B, and x, y, and b represent the amounts in proportional parts by which X, Y, and B exceed their nominal values, eq 33 may be written in the form

$$N(1+x)=N(1+y)(1+a-a_n)/(1+b). \qquad (34)$$

Expanding eq 34 and neglecting 3d and higher-order terms gives

$$x=y+(a-a_n)-b+y(a-a_n)-b(a-a_n)+b^2-by. \qquad (35)$$

Letting z represent that part of eq 35 which, for the series of measurements, is a constant, and, for the present, leaving out of consideration the second-order term $(y-b)(a-a_n)$ gives

$$\left.\begin{aligned} x_1&=a_1+z\\ x_2&=a_2+z\\ x_3&=a_3+z\\ x_4&=a_4+z\end{aligned}\right\} \qquad (36)$$

etc.

Since X_2 and X_4 are known, x_2 and x_4 are known. Furthermore, all a's will be read from the bridge. Therefore, z may be determined from the second or fourth of eq 36, or what is better, from both of these equations and the mean value used. Substituting the mean value of z in each of eq 36 gives a value for each of the x's and consequently for X_1, X_2, X_3, X_4, etc. in terms of the unit in which values of X_2 and X_4 are expressed.

To see to what extent the values thus found are in error as a result of the approximations involved in eq 36, it is necessary to consider the second-order term $(y-b)(a-a_n)$ of eq 35, the magnitudes of x_1, x_2, x_3, x_4, etc. and what constitutes nominal values of each of the bridge arms. It was intended that resistances of the sections of the A arm be so proportioned that differences of readings of the dials equal differences in resistance in parts per million of the resistance the A arm has when the reading of the dials is 5555. This resistance (instead of 25, 100, or 1,000 ohms, depending on what sections are used and the way they are connected) should be taken as the nominal value of A. The appropriate multiple of the unit used is taken as the

nominal value of the X's, and nominal values of B and Y are so chosen that

$$B_n/Y_n = A_n/X_n.$$

As a result of the way in which the eq 36 are used z includes all of $(y-b)(a-a_n)$ except the part equal to $(y-b)[a-(a_2+a_4)/2]$, while the nominal value chosen for A makes

$$y-b=5555-a+x, \text{ approximately.} \tag{37}$$

Therefore $(5555-a+x)\ [a-(a_2+a_4)/2]$ may be taken as the error in or a correction term for any one of eq 36. From the data obtained in a series of measurements it is therefore possible to determine, almost at a glance, the correction which, if significant, should be applied to one or more of eq 36. The first factor in this correction term is a constant for the series of measurements and depends upon the relative departures of Y and B from their nominal values. By a selection of the standard resistor to be used in the Y arm, usually it is possible to make this factor less than 0.02 percent. If the first factor is less than 0.02 percent and the resistance of no one of the standard resistors substituted one after the other in the X arm differs from the mean of X_2 and X_4 by more than 0.05 percent, in no case will the error or correction term exceed 1 part in 10 million.

Returning now to a further consideration of the sensitivity of bridges, the substitution method not only obviates the requirement of an accurate knowledge of the resistances of the A, B, and Y arms of the bridge, but, for the most part, eliminates the effects of heating by the test current in these arms. Therefore, if the battery and galvanometer are in their normal positions, it is permissible to use in the Y arm a standard resistor having a higher nominal value than those being substituted alternately in the X arm and thus realize a somewhat higher permissible sensitivity, as may be seen by reference to eq 26z. If, on the other hand, the positions of the battery and galvanometer are interchanged, the use in the Y arm of a standard of lower nominal value than those being substituted in the X arm gives a higher permissible sensitivity only in case W is larger than V, as may be seen by reference to eq 27x. Furthermore, it is not the load coefficients of the resistors being compared, but their differences or spread which limits the permissible potential drop in these resistors.

Equation 36, and others which follow under the heading "Bridge Equations," are applicable if the Wheatstone bridge method is used. They are also applicable if the Thomson bridge method is used, provided the adjustment is made in the manner which will be described later (or its equivalent), and provided further that the resistance of the potential lead which then constitutes a part of the A arm of the bridge is not so large but that differences in readings of the dials may be taken as differences in the ratio of A to B in parts per million of its nominal value. If x_1 is the resistance of this lead, including the adjustable rheostat (see fig. 9), the effect of this resistance is taken into account by the addition to the right-hand members of eq 36 the second-order term $(x_1/A_n)[a-(a_2+a_4)/2]$. Usually x_1/A_n does not exceed 0.05 percent, and $a-(a_2+a_4)/2$ does not exceed 0.05 percent, so usually this second-order term amounts to less than 1 part in 1 million.

A point which should be brought out here is that if a change in the unit of resistance is made, no change will be required either in the bridge or the procedure followed in the reduction of the observational data. A change in the unit will change x in the correction term

$$[5555-a+x][a-(a_2+a_4)/2],$$

but an approximately equal change in a, a_2, and a_4 may be made by the selection of a different standard resistor for use in the Y arm of the bridge.

2. INTERCHANGE METHOD

If two nominally equal resistances, X_1 and X_2, are to be compared, the bridge may be balanced by an adjustment of the A arm, first with X_1 in the X arm and X_2 in the Y arm and then with X_2 in the X arm and X_1 in the Y arm. This gives two relations, which may be written as follows:

$$X_1=X_2\frac{A_n}{B}(1+a_1-a_n). \tag{38}$$

$$X_2=X_1\frac{A_n}{B}(1+a_2-a_n). \tag{39}$$

From eq 38 and 39 it follows that

$$X_2=X_1\sqrt{1+a_2-a_1+(a_2-a_n)(a_n-a_1)+(a_1-a_n)^2} \tag{40}$$

If, therefore, conditions are such that the second-order terms under the radical and terms of the order of $(a_2-a_1)^2$ may be neglected,

$$X_2=X_1\,[1+(a_2-a_1)/2]. \tag{41}$$

If, in addition, X_1 has so nearly its nominal value that $x_1(a_2-a_1)/2$ may be neglected,

$$x_2=x_1+(a_2-a_1)/2, \tag{42}$$

where x_2 and x_1 are the departures of X_1 and X_2 from their nominal values.

3. KNOWN RATIO METHOD

In the comparison of two resistances, X_1 and X_2, whose nominal values differ by a factor such as 2, 3, 5, or 10, different procedures may be used, but most of these only in special cases. The following is almost universally applicable and most frequently used in this Bureau.

The ratio of the bridge is set nominally equal to the ratio of the resistances of the two standards, which are placed one in the X arm and the other in the Y arm. Then, if X_2 is known and it is placed in the Y arm of the bridge,

$$X_1=RX_2(1+a_1-a_r), \tag{43}$$

where R is the nominal ratio of X_1 to X_2, a_1 is the reading of the dials of the A arm of the bridge, and a_r is the reading (as yet undetermined)

of the dials of the A arm, for which the ratio of A to B has the nominal ratio of X_1 to X_2. From eq 43 it follows that

$$x_1 = x_2 + a_1 - a_r, \text{ approximately,} \tag{44}$$

where x_1 and x_2 are the departures of X_1 and X_2 from their nominal values.

If, however, X_2 is known and it is placed in the X arm of the bridge

$$x_1 = x_2 + a_r - a_1. \tag{45}$$

If A_n/B differs from the nominal ratio of X_1 to X_2 (or X_2 to X_1) by less than 3 parts in 10,000, and X_1 and X_2 differ from their nominal valute by less than 3 parts in 10,000, the approximations are not likely so exceed 1 part in 10 million. The procedures followed in determining a_r will be considered in the next section.

VIII. ESTABLISHMENT OF KNOWN RATIOS

The principle underlying one of the procedures followed in finding the reading of the A arm (that is, a_r of eq 43) of the bridge for which the ratio of A to B is accurately an integer may be illustrated as follows. If there are at hand n standards or coils having nearly equal values, M_1, M_2, M_3, etc., and S is their resistance when connected in series,

$$S = M_1 + M_2 + M_3 + \text{etc.} \tag{46}$$

Also, if P is their resistance when connected in parallel,

$$\frac{1}{P} = \frac{1}{M_1} + \frac{1}{M_2} + \frac{1}{M_3} + \text{etc.} \tag{47}$$

Now, let M be their mean resistance and m_1, m_2, m_3, etc. be the departures of each in proportional parts from the mean of all. Then

$$S = M(1 + m_1 + 1 + m_2 + 1 + m_3 + \text{etc.}) \tag{48}$$

and

$$\frac{1}{P} = \frac{1}{M(1+m_1)} + \frac{1}{M(1+m_2)} + \frac{1}{M(1+m_3)} + \text{etc.} \tag{49}$$

Assuming that the resistances of the standards or coils are so nearly equal that the third and higher powers of m may be neglected, expansion by the binomial theorem of the terms $1/(1+m_1)$, $1/(1+m_2)$, $1/(1+m_3)$, etc. in eq 49 gives

$$\frac{1}{P} = \frac{1}{M}(1 - m_1 + m_1^2 + 1 - m_2 + m_2^2 + 1 - m_3 + m_3^2 + \text{etc.}). \tag{50}$$

Since by definition $m_1 + m_2 + m_3 + \text{etc.}$ equals zero,

$$S = nM, \tag{51}$$

and

$$\frac{1}{P}=\frac{n}{M}\left(1+\frac{1}{n}\sum m^2\right), \tag{52}$$

where $\Sigma m^2=m_1{}^2+m_2{}^2+m_3{}^2+\text{etc.}$ It follows, therefore, from eq 51 and 52 that

$$\frac{S}{P}=n^2\left(1+\frac{1}{n}\sum m^2\right). \tag{53}$$

There is no difficulty in adjusting a group of standard resistors or coils so that the resistance of no one differs from the mean of all by more than a few, or even 1 part in 10,000, in which case $\Sigma m^2/n$ would be not more than 1 part in 10 million or 1 part in 100 million, so usually $\Sigma m^2/n$ may be neglected. However, the conductors used in making the series and parallel connections will in general have resistances which cannot be neglected.

Presumably Lord Rayleigh was the first to use the same coils connected alternately in parallel and in series [72] in building up from unit standards. Steps of 4, 9, 16, 25, etc. are readily obtained simply by changing the connections of the appropriate numbers of coils of approximately equal resistances from parallel to series. Unfortunately, the square root of 10 is not an integer, so a step of 10, which is most frequently needed, cannot be obtained directly in this way.

In this Bureau an auxiliary apparatus constructed in 1912 is used for finding the reading of the dial switches of the A arm of the bridge for which the ratio of A to B is accurately 10 to 1. The circuit of this apparatus is shown in figure 8. It will be observed that there are seven resistance sections, six of 150 ohms each, and one of 50 ohms, all connected in series. Two amalgamated copper links not shown in the figure serve to connect either group of three of the 150-ohm sections in parallel. Also other amalgamated copper links are provided for use in determining the relative resistances of the sections. The resistances of the links are relatively low and definite and the arrangement is such that the resistance of each link, including two amalgamated contacts and the appropriate portion of each of two terminal blocks may readily be measured. In the appendix on terminals and contacts, it will be shown that the effect of the resistances of parts of terminal blocks not included as parts of links or as parts of resistance sections is negligibly small. All other connectors are so arranged that the effect of their resistance is eliminated in the procedure followed in the use of the apparatus.

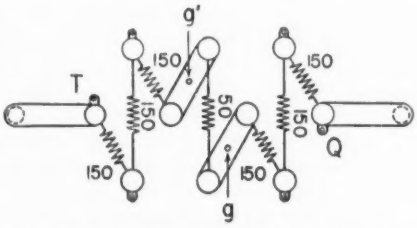

FIGURE 8.—*Circuit diagram of the auxiliary apparatus used for determining the reading of the dial switches of the NBS precision bridge, for which the ratio of the resistance of the A arm to the resistance of the B arm is 10 to 1.*

When this apparatus is placed in the bridge with a galvanometer connection made at g, it constitutes the X and Y arms of the bridge. With the three sections on the right connected in parallel and all other sections in series, the ratio of the resistance of the series sections

to the resistance of the parallel sections, that is, the ratio of X to Y, is very nearly 10 to 1. With A set at a nominal value of 100 ohms, B set at a nominal value of 10 ohms, and with no part of the auxiliary apparatus in the A or B arms, the bridge is balanced by an adjustment of the A arm and the reading of the dial switches recorded. Then with the battery leads transferred to the points T and Q, the bridge is again balanced by an adjustment of the A arm and the reading of the dial switches recorded.

In the first balance, the resistance of the connection between the normal junction of the X and A arms and the point T is in the X arm, while in the second balance this resistance is in the A arm. Likewise, in the first balance the resistance between the normal junction of Y and B and the point Q is in the Y arm, while in the second it is in the B arm. Since the ratio of A to X and B to Y is 1 to 5, one-sixth of the difference between the two readings [111] of the dial switches applied to the first in the direction which moves it towards the second corrects for the error which otherwise would result from the resistances of the end connectors of the auxiliary apparatus. Now the auxiliary apparatus is turned through 180°, the parallel connectors are transferred to what is now the right side, and the galvanometer connection is made at g', so as to again place the 50-ohm section in the X arm of the bridge. Then two additional balances of the bridge are established and weighted as just described, giving a second reading of the dial switches, corrected for the effect of the resistances of the end connectors. Taking the mean of these two readings corrects for the difference in the mean resistance of the three sections on the right and the three sections on the left, as shown in figure 8. It is therefore the reading of the dial switches for which the ratio of A to B is accurately 10 to 1, excepting a correction to account for the lack of strictly proper adjustment of the 50-ohm section and a further correction to account for the resistances of the paralleling connectors. The first of these is -0.1, the amount expressed in proportional parts by which the resistance of the 50-ohm section exceeds the mean resistance of the other six sections taken three at a time in parallel, and corrected for the resistances of the paralleling connections, while the second is $+8/9$ the mean resistance of the paralleling connections expressed in proportional parts of 50 ohms. These corrections (which amount to only a few parts in 1 million and are easily determined to 1 part in 10 million) applied to the mean of the two readings, referred to above, give the reading a_r of eq 43, 44, and 45, corresponding to a ratio of A to B equal to 10 to 1, with a probable error not in excess of 1 part in 3 million. However, to obtain this precision requires interpolation from galvanometer deflections, since the apparatus reads directly only to 1 part in 1 million. The reading of the dial switches of the A arm of the bridge for which the ratio of A to B is 10 to 1 must be known to a high accuracy, since (starting with 1-ohm standards, which are used in maintaining the unit) any error in this reading enters once in the evaluation of the 10-ohm standards, twice in the evaluation of the 100-ohm standards, three times in the evaluation of the 1,000-ohm standards, and four times in the evaluation of the 10,000-ohm standards. It also enters once in the evaluation of the 0.1-ohm standards, twice in the evaluation of the 0.01-ohm standards, three times in the evaluation of the 0.001-ohm standards, and four times in the evaluation of the 0.0001-ohm standards.

When it is desired to find the reading of the A arm for which the ratio of A to B is 2 to 1, the procedure is as follows:

1. The A arm is connected so that its nominal resistance is 100 ohms, the B arm is connected so that its nominal resistance is 50 ohms, the X arm is arranged to receive two standard resistors connected in series, and the Y arm is arranged to receive one standard resistor.

2. Three standard resistors having the same nominal value (preferably 100 ohms) are used.

3. The bridge is balanced by an adjustment of the A arm with each of the three standard resistors placed one after the other in the Y arm, the other two being in the X arm.

4. The mean of the three readings of the A arm, after a correction is applied to account for the resistance of connectors, is taken as the reading of the A arm for which the ratio of the resistance of the A arm to the resistance of the B arm is 2 to 1.

A similar procedure is followed in determining the reading of the A arm for which the ratio of A to B is 3 to 2, 3 to 1, 2 to 3, 1 to 3, 1 to 2, or involves other small integers.

IX. ADJUSTMENTS OF THOMSON BRIDGE

A simplified diagram of the bridge circuit when the Thomson bridge method is used, is shown in figure 9. Here, x_1 represents one of the

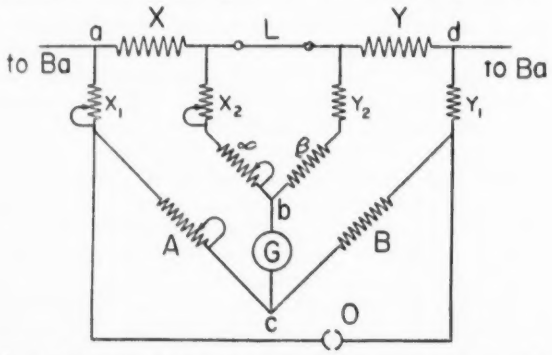

FIGURE 9.—*Circuit diagram of the Thomson bridge arranged for explanation of the procedure followed in establishing balances of the bridge in such a way as to make* $A/B=X/Y$.

potential terminals of X, the potential lead, and a rheostat continuously adjustable through a range of about 0.01 ohm; y_1 represents one of the potential terminals of Y, the potential lead, and a resistance which for the purpose at hand may be considered as fixed, though actually it is adjustable in steps; x_2 is similar to x_1; and y_2 is similar to y_1. L and O represent connectors of rather low resistance, which may be opened or closed at will.

The adjustments are carried out as follows:

(1) With the connector L closed and the connector O open, the direct-reading arms, A and α are so adjusted as to give a balance of the bridge. This makes

$$\frac{A+x_1}{B+y_1}=\frac{X}{Y} \text{ approximately.} \tag{54}$$

(2) With the connectors L and O both closed, the bridge is balanced by an adjustment of x_1. These two adjustments make

$$\frac{x_1}{y_1}=\frac{X}{Y} \text{ approximately.} \tag{55}$$

(3) With the connectors L and O both open, the bridge is balanced by an adjustment of x_2 [74]. This makes

$$\frac{X+x_2+\alpha}{Y+y_2+\beta}=\frac{A+x_1}{B+y_1}. \tag{55a}$$

(4) With L closed and O open, the bridge is again balanced by an adjustment of the direct-reading arms which, as stated above, are mechanically connected so always read the same.

These four adjustments give

$$\frac{X}{Y}=\frac{A}{B}, \tag{56}$$

usually to the precision desired or attainable. The process is really one of successive approximations, but the observer soon learns to judge from the magnitude of the change in the reading of the dials in making the fourth adjustment and the known precision required in the various adjustments, if the adjustments need be repeated. Equation 56 is the same as the Wheatstone bridge equation. Therefore, in making a series of measurements, the data recorded and all calculations are the same as though the Wheatstone bridge method had been used. That is, only the readings of the dials for each of the final balances of the bridge are recorded, and these are substituted in the appropriate equation 36, 42, 44, or 45 considered above.

Since x_1/A and y_1/B seldom exceed 0.001, while in the final adjustment x_1 is in series with A and y_1 is in series with B, the precision required in the second adjustment may be two or three orders lower than that sought in the measurement. Furthermore, the precision required in the third adjustment need not be high, and in many cases this adjustment may be omitted. An important feature of the procedure is that the adjustments result in making $X/Y=A/B$, whether or not the four-terminal conductors, X and Y, are linear.

The simplicity of the adjustments results more from having suitably designed and well-constructed apparatus than from the procedure followed, which differs only in a minor detail from that described by Jaeger and Diesselhorst [45]. Procedures for determining the correction terms [P] of the Thomson bridge equation or for making these terms negligibly small have been discussed by several of those who have had occasion to use the Thomson bridge method.

X. PRECISION AND ACCURACY

The expected precision of the measurements may not be fully realized on account of slightly faulty insulation in the bridge or in one of the resistances being measured, static electrical effects, rapidly varying thermoelectromotive forces, etc. Whether or not it is actually obtained depends to a large extent on the temperament and skill of the observer. To be properly qualified he should have a desire to do

the job well, be neither easily fatigued nor perturbed, recognize slight disturbances promptly and be able to locate their source and correct the difficulty, and, above all else, be able to differentiate between that which is essential and that which is not essential in each of eight orders of magnitudes.

At present the resistance of no standard resistor can be presumed to be known (either in international ohms [3] or in absolute ohms) to an accuracy within one order of the precision to which two or more nominally equal resistances can be compared (if within the range considered here) or quite to the precision to which an 0.0001-ohm standard may be compared with a 10,000-ohm standard (provided a sufficient number of intermediate steps is used). It might seem, therefore, that the precision considered here is unnecessarily high. However, the precision of the resistance measurements is a factor contributing somewhat to the accuracy to which units of resistance may be realized and is an important factor in selecting the particular standard resistors used in maintaining a unit of resistance. For judging the relative quality of standard resistors, a precision of 1 part in 1 million is really needed, and it is a great convenience that for the most part this is readily obtained. As an indication of a limited need for a somewhat higher precision, it may be pointed out that the resistance of each of a fairly large group of 1-ohm standard resistors (of the double-walled type constructed by Thomas in 1933) is remaining so constant relative to their mean resistance that only by making the comparisons to a precision of about 1 part in 10 million can the changes occurring during a few months be detected.

In the above discussions a stated precision represents twice the probable error of a single measurement. The precision is estimated from the extent of the agreement of results on repetitions of the measurement under different conditions, or the consistency of results obtained in case more than the minimum required number of measurements is made.

For example, if five 1-ohm standards are to be intercompared, they may be substituted one after another in the same arm of the bridge. The five balances of the bridge thus obtained give data from which the resistance of each standard may be calculated in terms of the resistance of any one or the mean resistance of any two or more. Repetitions of these measurements under conditions giving different distributions of the systematic errors furnish data for determining the probable error of a single measurement by the substitution method, that is, the probable error of each of the five results obtained from five balances of the bridge.

The same five standards may be compared by interchange between the X and Y arms of the bridge in all possible combinations. The data obtained from the 20 balances of the bridge then serve in calculating the resistance of each standard in terms of the resistance of any one or the mean resistance of any two or more, the probable error in the results and the probable error in a single measurement by the interchange method. If all balances of the bridge were equally reliable, the probable error in the results obtained from the 20 balances of the bridge would be smaller than that obtained from five balances

[3] Here by "international ohm" is meant a unit of resistance realized by the use of a column of mercury under specified conditions, as distinguished from the unit of resistance of this country or the mean of the units of resistance of several countries.

of the bridge; also the probable error of a single measurement by the interchange method would be smaller than by the substitution method.

In the course of time many sets of measurements are made and repeated, so fairly definite estimates may be made of the precision obtained in individual measurements under various conditions. In repetitions of measurements after one or more days, the agreement of results occasionally is not as good as would be expected from the estimated precision of the measurements. In such cases, extending the measurements over longer periods of time usually leads to the conclusion that changes in the standard resistors are mainly responsible for the lack of agreement of results. Consequently, there is little increase in the accuracy of the measurements other than the detection of a possible error in recording a bridge reading, by making more than the minimum required number of balances of the bridge. Therefore, in comparing a number of standard resistors, usually this is done by the substitution method, making the minimum required number of balances of the bridge, and then repeating this series of measurements one or more days later, under conditions giving different readings of the A arm of the bridge. In case the results obtained from the two series of measurements are as consistent as should have been expected, means of the values thus obtained are taken as the results of the measurements. In case of a lack of a reasonable agreement in the results, further measurements are made to determine the cause and to remedy the difficulty should it be found to be a defect in the measuring apparatus instead of changes in the resistance of one or more of the standard resistors used in the series of measurements. In certificates for standard resistors, usually the stated accuracy of the values given is determined not by the precision of the measurements but by the uncertainty of the values in international ohms of the standard resistors used in maintaining units of resistance and the estimated changes with time of the resistances of the standard resistors being certified. More frequently than otherwise, the stated accuracy is 0.005 percent.

XI. APPENDICES

In the above discussion a number of important points have been passed over with little or no consideration. Some of these will now be considered somewhat in detail under the headings: Terminals and Contacts; Thermoelectromotive Forces; Insulation; Optical System; Methods of Analyses; Ohm's Law; and Units of Resistance; while others will be found discussed in publications to which reference is made.

APPENDIX 1. TERMINALS AND CONTACTS

(a) STANDARD RESISTORS

For the resistance of a conductor to be definite, one of several requirements is that the current always enter and leave the conductor in such a way as to give always the same or an equivalent distribution of the current density in that part of the conductor between the particular equipotential surfaces which serve in limiting or defining the resistance. In addition, the potential drop must always be taken between the same two or equivalent equipotential surfaces. Each time a standard resistor is removed from and replaced in a circuit in which there is an electromotive force some change occurs in the current distribution over the surfaces through which the current enters and leaves the standard, and the potential drop cannot always be taken between exactly the same two or equivalent equipotential surfaces. It will therefore be of interest to see what conditions are neces-

sary in order that the resistance be definite to 1 part in 1 million, considering only the effects of contacts and current and potential distributions in terminals. But first it will be necessary to state more precisely what is to be understood by the resistance, or rather the resistances, of a conductor.

Since the more important resistances of a four-terminal conductor are more definite than is the resistance of a two-terminal conductor, four-terminal conductors will be considered first. Referring to figure 10, the heavy lines designated 1, 2, 3, and 4 will be considered as representing surfaces on which connections to other conductors may be made, that is, they will represent terminals. In case 1 and 4 are normally used as current terminals and 2 and 3 are normally used as potential terminals, the resistance may be defined as the ratio of the drop in potential from 2 to 3 to the current entering on 1 and leaving on 4. This will be designated the resistance (1234). In case 2 and 4 are used as current terminals and 1 and 3 as potential terminals, the resistance may be defined as the ratio of a drop in potential from 1 to 3 to the current entering on 2 and leaving on 4. This will be designated the resistance (2134).

It will thus be seen that a four-terminal conductor has as many four-terminal resistances as there are permutations of four numbers. However, only two of these resistances are independent of each other, and the relations between all are rather simple. Since there are so many resistances, it is a convenience to divide them into groups and to attach distinctive names to these groups. The four equal resistances (1234), (2143), (4321), and (3412) will be called the direct resistance. The four equal resistances (1432), (4123), (2341), and (3214) will be called the cross resistance, and the four equal resistances (1243), (2134), (3421), and (4312) will be called the diagonal resistance. When defined in this way, the direct resistance minus the cross resistance equals the diagonal resistance, as was pointed out in a previous paper [101]. Furthermore, any one of the 12 remaining four-terminal resistances is equal in magnitude but opposite in sign to either the direct, the cross, or the diagonal resistance.

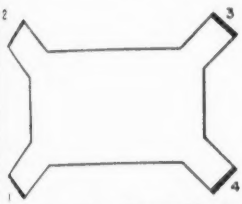

FIGURE 10.—*Generalized four-terminal conductor.*

It is also a convenience to divide four-terminal conductors into groups in accordance with the relative magnitudes of the resistances and attach names to these groups. Conductors for which the cross resistance is less than a millionth part of the direct resistance will be called linear,[4] and all others will be called nonlinear.

If variations in the current and potential distribution on terminals are to affect the direct resistance by certainly less than 1 part in a million, the terminals must be so designed that the sum of the following potential ratios will be less than 0.000 001. With 1 and 4 used as current terminals, (1) the potential difference between any two points on 2 to the potential difference between 2 and 3; (2) the potential difference between any two points on 3 to the potential difference between 2 and 3, and with 2 and 3, used as current terminals; (3) the potential difference between any two points on 1 to the potential difference between 1 and 4; and (4) the potential difference between any two points on 4 to the potential difference between 1 and 4. This condition is generally realized in standard resistors designed for precise measurements, but is seldom realized in standard resistors designed for use with large currents. In many cases the resistance depends upon the manner of connecting the current leads to the current terminals to the extent of 0.01 percent, and in exceptional cases to the extent of 1 percent.

The resistance of a two-terminal resistor may be defined as the ratio of the potential drop to the current, taken between two equipotential surfaces lying partially in the terminals of the resistor and partially in the conductors through which the current enters and leaves the resistor. From an experimental standpoint, it is better to define the resistance of a two-terminal resistor as the difference in resistance of two four-terminal conductors which are the same in all respects, except that one does and the other does not include the two-terminal resistor.

[4] It should be noted that the distinction made here between a linear and nonlinear conductor does not involve a proportionality and lack of proportionality of the potential drop to the current.

To illustrate what is meant by this statement and to show that the resistance so defined is not strictly a constant, let it be assumed that there is at hand a short copper rod 1 cm in diameter and having plane end surfaces which are amalgamated and that the resistance of this rod is to be measured. Also let it be assumed that there are at hand two sets of cylindrical copper terminal blocks having plane amalgamated end surfaces, the diameter of one set of blocks being 1 cm and the diameter of the other set of blocks being 2 cm. Each terminal block has a flexible current lead and a flexible potential lead. If (1) the copper rod is placed between the amalgamated surfaces of the terminal blocks having the smaller diameters and the resistance of the four-terminal conductor thus constituted is measured; if (2) the copper rod is removed, the amalgamated surfaces of terminal blocks are placed in contact, and the resistance of the four-terminal conductor thus constituted is measured; and if (3) the latter measured value is subtracted from the former measured value, a value is obtained for the resistance, as defined, of the copper rod. This resistance will in general be slightly higher than that which may be calculated from the known resistivity and dimensions of the copper rod. Now, if this procedure is repeated, using the terminal blocks having the larger diameter, a still higher value will be obtained for the resistance of the copper rod, provided the contacts between the rod and terminals are equally good in the two cases, and provided also that when terminal blocks are placed together, the contact is such as to give a substantially uniform current density over the contact area. In the first case, one amalgamated contact is included in the measured resistance. In the second case, there is in addition a nonuniformity of current density in the vicinity of the contacts, which in effect increases the measured resistance.

Obviously, therefore, the resistance of a two-terminal standard resistor depends to a greater or less extent on the manner in which it is connected into a circuit. For example, when a two-terminal standard resistor is placed in an arm of a Wheatstone bridge, the bridge arm, which is a four-terminal conductor, consists of the standard and the end connectors. If the terminals of the standard and of the end connectors are amalgamated, the end connectors consist of two amalgamated surfaces or mercury cups and two terminal blocks of the bridge. With end connectors of the usual type, that is, having deep mercury cups, the resistance of the bridge arm may depend on the amount of the mercury in the cup and the position of the terminals of the standard in the cup to the extent of a few microhms. This difficulty is obviated to a considerable degree by the use of plane amalgamated surfaces, instead of mercury cups.

What is of more importance than the form of the contact surfaces is the gradual accumulation of copper amalgam in the solid phase on the terminals of standard resistors and on end connectors or contact blocks. Cases have been observed in which the removal of the amalgam in the solid phase has resulted in a lowering of the resistance by more than 10 microhms (though a definite reason for so large a change has not been found). Even when the amalgamated surfaces of the terminals of the standard resistor and of the terminal blocks with which it may be used are apparently in good condition, the resistance may be expected to depend on the resistances of the amalgamated contacts and the distribution of the current density in their vicinity to the extent of a few microhms.

(b) TERMINAL BLOCKS OF BRIDGE

The more important terminal blocks of the NBS precision bridge and all terminal blocks of the auxiliary apparatus used in establishing known resistance ratios have four or more terminals.

A diagram of a main terminal block of the bridge which is used when the *A* arm is 25 or 100 ohms is shown in figure 11 (a). The block is constructed of copper, and terminals 1, 2, 4, and 6 are copper wires soldered in holes drilled in the under side of the block. Terminal 3 is a plane amalgamated surface (not a mercury cup), and terminal 5 is a small mercury cup. Terminals 1 and 5 are so located with respect to the other terminals that they may be considered as equivalent, unless both are used at the same time. Terminal 6 is connected to the 900-ohm section of the *A* arm. The main terminal block normally used with the *B* arm of the bridge, when this arm has a resistance of 25 ohms or 10 ohms, is similar to that shown in figure 11 (a) except that it has no terminal 6. When the Wheatstone bridge method is used, terminals 1 (or 5), 3 and 4 are used as shown in figure 11 (b). When the Thomson bridge method is used, terminals 1, 2, and 4 are used as shown in figure 11 (c). Consequently, the current distributions in these blocks are not the same in the two cases, and as a result the contributions of one terminal

block to the resistance of the A arm and of the other terminal block to the resistance of the B arm are not the same. When the Thomson bridge method is used the resistance contributed to the A arm or to the B arm by its terminal block is higher than when the Wheatstone bridge method is used, by the four-terminal resistance (1234).

Since the reading of the A arm of the bridge for which the ratio of A to B is 10 to 1 is determined by the Wheatstone bridge method and this reading is used with both the Wheatstone and the Thomson bridge methods, the effect of these resistances should be known. Measurements of these resistances give values ranging from -0.3 to $+0.3$ microhm, depending upon the point on the amalgamated

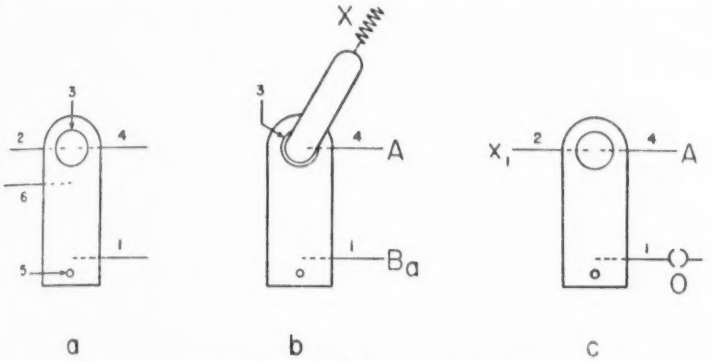

FIGURE 11.—*A main terminal block of the NBS precision bridge.*

a shows the six terminals of the terminal block, *b* shows the three terminals used with the Wheatstone bridge method, and *c* shows the three terminals used with the Thomson bridge method.

surface serving as terminal 3. However, as used, the connection is distributed somewhat uniformly over the central portion of the surface, so probable values lie in the range from -0.1 to $+0.1$ microhm. Therefore, since the lowest resistance used in the A arm is 25 ohms and the lowest resistance used in the B arm is 10 ohms, changes in the current distribution resulting from a change from one bridge method to another and from variations in contacts on the amalgamated surfaces affect the ratio of A to B only a few parts in 100 million.

The contribution of these terminal blocks to the resistances x_1 and y_1 when the Thomson bridge method is used is of no consequence, since adjustments make $(A+x_1)/(B+y_1)=(A/B)=(X/Y)$. In addition to the resistance of an amalgamated surface, these terminal blocks contribute approximately 1 microhm to the resistance of the X arm and Y arm of the bridge when the Wheatstone bridge method is used. However, if a precision of 1 part in 1 million is expected, the resistances of the X and Y arms must be 10 ohms or more, or the conductors must be of the four-terminal type. When the conductors are of the four-terminal type, all connecting conductors can be changed from one to another of two adjacent arms of the bridge [111], and this makes it possible to apply a correction to account for the resistances of connectors, including amalgamated contacts.

(c) TERMINAL BLOCKS OF AUXILIARY APPARATUS

In the previous discussion of the auxiliary apparatus, used in determining the reading of the dial switches of the A arm of the bridge for which the ratio of A to B is 10 to 1, very little was said concerning corrections to account for the effect of the resistances of the terminal blocks. This matter will now be considered in detail.

Each of these terminal blocks is a four-terminal conductor and consequently has two independent four-terminal resistances. One of the end terminal blocks is shown in elevation in figure 12 and in plan in figure 8. Here terminal 3 is an amalgamated surface on which a copper link rests when the three resistance sections adjacent to the terminal block are connected in parallel, terminal 4 is a part of one of the 150-ohm resistance sections, terminal 2 is a small mercury cup which is used as a potential (or current) terminal in measuring the resistance of the paralleling connection, that is, of the copper link, the two amalgamated con-

tacts and a portion of each of two terminal blocks. Terminal 2 is also used as a current or potential terminal, both when the three 150-ohm resistance sections adjacent to it are connected in series and when they are connected in parallel. Terminal 1 serves in connecting the apparatus into the bridge. With the parallel connection two-thirds of the current is through terminal 3, one-third of the current is through terminal 4, and all of the current is through terminal 2 or terminal 1. In measuring the resistance of the paralleling connection, all of the current is through terminal 3 and all through terminal 1 or terminal 2. With the series connection, all the current is through terminal 1 or terminal 2, and all through terminal 4.

One of the terminal blocks to which two 150-ohm sections are connected is shown in elevation in figure 13 and in plan in figure 8. Here terminal 3 is an amalgamated surface on which a copper link rests when the resistance sections terminating in the block are connected in parallel, and 2 is a small mercury cup used as a terminal in measuring the resistance of the paralleling connection.

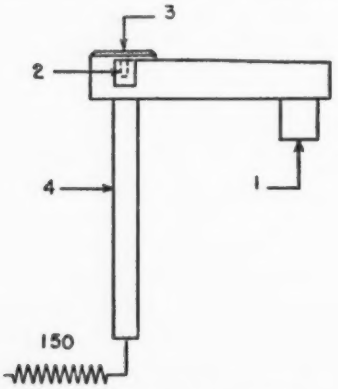

FIGURE 12.—*An end terminal block of auxiliary apparatus.*

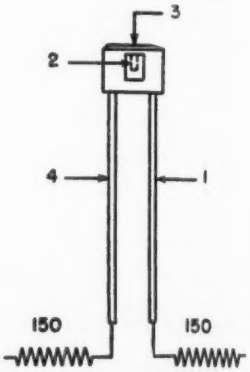

FIGURE 13.—*A terminal block of auxiliary apparatus to which two 150-ohm resistance sections are connected.*

Terminal blocks of this type are also used with three different current distributions. Consequently, the effects of the four-terminal resistances of these terminal blocks are more complicated than the effects of the four-terminal resistances of the main terminal blocks of the bridge, in which it was necessary to consider only two current distributions in each block.

In the use of the auxiliary apparatus, what is of most importance is the ratio of the four-terminal resistances of each of the two groups of three 150-ohm resistance sections when connected in series, R_s, to their four-terminal resistances when connected in parallel, R_p. The effect of slight inequalities in the resistances of the 150-ohm sections of each group has already been discussed, and here it will be assumed that this effect is too small to require consideration. Since the problem is complicated, only the results which have been obtained will be given here, while the analysis will be given in the appendix on methods of analyses. With the notation shown in figure 14, the ratio

$$R_s/R_p=9\{1-4[L_1+L_2+(1234)_a-(1432)_a/2-(1234)_b+(1432)_b-(1234)_e+(1432)_e+(1234)_d-(1432)_d/2]/450\} \quad (57)$$

Also the resistance of terminal 1 of terminal block a, that is, the potential drop from 1 to 2 of a to the current entering on 1 of a and leaving on 1 of d, and which will be designated the resistance $(1_a1_a2_a1_d)$, is higher by two-thirds the resistance $(1432)_a$, and the resistance of terminal 1 of terminal block d, $(1_d1_d2_d1_a)$, is higher by two-thirds the resistance $(1432)_d$ with the parallel connection than with the series connection. The former is taken into account by the procedure followed in the use of the apparatus. However, the latter represents the increase in resist-

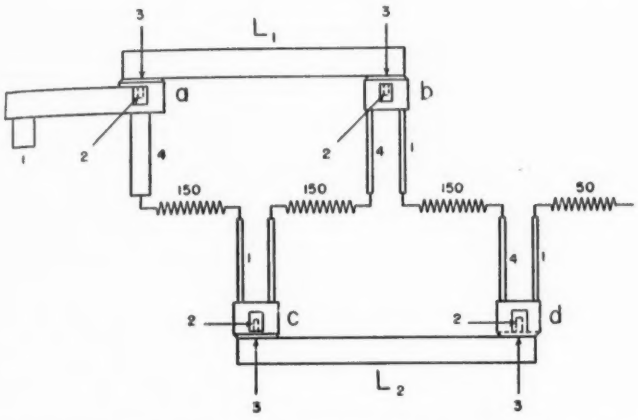

FIGURE 14.—*Parallel connection of three 150-ohm resistance sections of the auxiliary apparatus.*

This connection involves the four terminal blocks, disignated *a*, *b*, *c*, and *d*, and two paralleling connectors, designated L_1 and L_2. The series connection is made by removing the paralleling connectors.

ance of the 50-ohm section resulting from the change of the connection of three 150-ohm sections from series to parallel. In eq 57, L_1 represents the four-terminal resistance of the paralleling connection between terminal blocks *a* and *b*, using 1 and 2 of terminal block *a*, and 1 and 2 of terminal block *b*, as the terminals (the measurement being made with the other paralleling connection open). A similar statement applies to L_2, and $(1234)_a$ represents the direct resistance of terminal block *a*, $(1432)_a$ the cross resistance of terminal block *a*, etc. A similar solution applies to the other group of three 150-ohm sections. To this point, three different current distributions in each of the eight terminal blocks of the auxiliary apparatus have been considered.

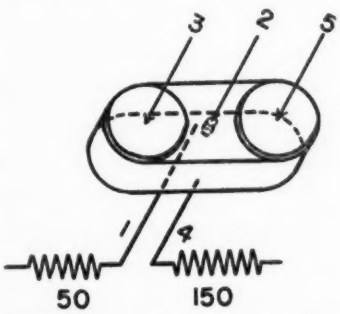

FIGURE 15.—*A central terminal block of the auxiliary apparatus.*

However, the relative values of resistance sections must be determined, and measurements of these require additional current distributions in at least some of the terminal blocks. Of these additional current distributions, the two in each of the central terminal blocks, *d* and *d'*, which are of the type shown in figure 15, are of most importance. The additional current distributions in the central terminal blocks are involved in a determination of the ratio of the resistance of the 50-ohm section to the resistance of the six 150-ohm sections connected in series. The measurements used in this determination consist in comparisons of the resistance of the 50-ohm section with the resistance of one and then of the other of the groups of three 150-ohm sections connected in parallel.

In the measurement of the resistance of one group of three 150-ohm sections connected in parallel, 1 and 2 of terminal block *a* and 5 and 2 of terminal block *d* are used as the four terminals. In the measurement of the resistance of the other group of three 150-ohm sections connected in parallel, 1 and 2 of terminal block *a'* and 5 and 2 of terminal block d' are used as the four terminals. These resistances will be referred to as the resistances $(1_a2_a2_d5_d)_p$ and $(1_{a'}2_{a'}2_{d'}5_{d'})_p$. The resistance of the 50-ohm section is measured, using 5 and 2 of terminal block *d* and 5 and 2 of terminal block *d'* as the four terminals. This will be referred to as the resistance $(5_d2_d2_{d'}5_{d'})$. The addition of the resistance $(2_a5_d1_d2_d)_p$ to the resistance $(1_a2_a2_d5_d)_p$ gives the resistance $(1_a2_a2_d1_d)_p$, which, when multiplied by the ratio given by eq 57, gives the resistance $(1_a2_a2_d1_d)_s$, that is, the four-terminal resistance of the three 150-ohm sections in series and with the normal distribution

of current in each of the four-terminal blocks involved. The resistances of the other group of three 150-ohm sections in series is obtained in the same way. The addition of the resistances $(2_d4_d5_d2_{d'})$ and $(2_{d'}4_{d'}5_{d'}2_d)$ to the resistance $(5_d2_d2_{d'}5_{d'})$, the measured resistance of the 50-ohm section, gives the resistance $(4_d2_d2_{d'}4_{d'})$, that is, the resistance of the 50-ohm section with what may be considered as the normal distribution of current in the its terminal blocks, d and d'. The result of these measurements may be expressed in the form

$$\frac{18R_{50}}{R_{150}+R_{150}+R_{150}+R_{150}+R_{150}+R_{150}}=1+a. \tag{58}$$

Here a is the amount, expressed in proportional parts, by which the resistance of the 50-ohm section exceeds the resistance of the six 150-ohm sections in series divided by 18, with what may be considered as the normal current distribution in all terminal blocks. As has previously been pointed out, $-a/10$ is a correction which must be applied in the use of the apparatus.

In the design of these terminal blocks and in the selections of values for the resistances of sections, an effort was made to make the effects of the different current distributions in the terminal blocks negligibly small. The extent to which this aim was realized may be seen from the relations stated above and the fact that measurements of the four-terminal resistances of the terminal blocks give values for each ranging from about -0.5 to $+0.5$ microhm, depending mainly on the part of the amalgamated surface serving as terminal 3. However, in use, each connection is distributed somewhat uniformly over the entire amalgamated surface, and under these conditions it is hardly probable that any of the four-terminal resistances lie outside the range from -0.3 to $+0.3$ microhm. Presumably, therefore, changes in current distribution in terminal blocks and systematic errors in the determination of the resistances of paralleling connections of the auxiliary apparatus contribute not more than 1 part in 50 million to the uncertainty of the reading of the A arm of the bridge, for which the ratio of A to B is 10 to 1.

Consequently, measurements of the relative values of the resistance sections and of the parelleling connections of the auxiliary apparatus may be made with any connection found convenient, provided only that each resistance section and each paralleling connection is considered as a four-terminal conductor.

What is of more importance than the resistances of the terminal blocks is the resistance of the amalgamated contacts included in the paralleling connections. Each of these contacts has a surface on the terminal block and another surface on the connecting link. The surfaces of the terminal blocks lie substantially in the same plane, and each has an area of approximately 1.25 cm². The under side of each connecting link is a plane, and the dimensions are such that when two terminal blocks are connected by a link, the entire contact areas of each of the two terminal blocks are effective. A few weeks, at most, previous to the use of the apparatus, the surfaces of terminal blocks and connecting links are polished with fine emery paper backed by a piece of plate glass or other plane surface having an area of not less than 300 cm² until the copper may be seen distinctly, indicating the removal of practically all of the accumulated copper amalgam in the solid phase. The surfaces are then reamalgamated, using the sodium-amalgam process. In placing links on the terminal blocks, they are moved back and forth parallel to the amalgamated surfaces until they slide freely indicating the removal from between the surfaces of any loose solid copper-amalgam or other solid material. With these precautions, a link may be removed and replaced by another of the same dimensions without changing the resistance of the paralleling connection by more than a few tenths of a microhm. In addition, the measured resistance of each parallel connector is in agreement with the value estimated from the dimensions and the resistivity of copper. The liberal design of these contacts, the possibility of removing the copper-amalgam in the solid phase without impairing the mechanical fit, and the exceptional precautions taken in their use result in a very much better performance than is realized with the contacts between standard resistors and terminal blocks of the NBS precision bridge.

Furthermore, if the average resistance of all contacts used in the paralleling connections were 1 microhm higher, or lower, at the time of use than at the time of measurement, this would result in an error in the determination of the reading of the A arm of the bridge, for which the ratio of A to B is 10 to 1, of less than 1 in 25 million. Consequently, it is not necessary that the resistances of the paralleling connectors be measured when and as used.

(d) THOMSON BRIDGE

When the Thomson bridge method is used with the battery and galvanometer in their normal positions there is current through each of the four terminals of each of the two four-terminal conductors designated X and Y in figures 9 and 16. There may therefore be some question as to which if any of the four-terminal resistances of X and Y are compared.

If the bridge is balanced according to the procedure outlined above or any other giving the same result, the connection through L may be opened or closed without changing the balance of the bridge; also the connection through O may be opened or closed without changing the balance of the bridge. That is, the balance of the bridge is independent of the resistance of the branch L and independent of the resistance of the branch O. As will be shown in the appendix on methods of analysis, a conductor having a resistance, R, and in which there is a current, I, may

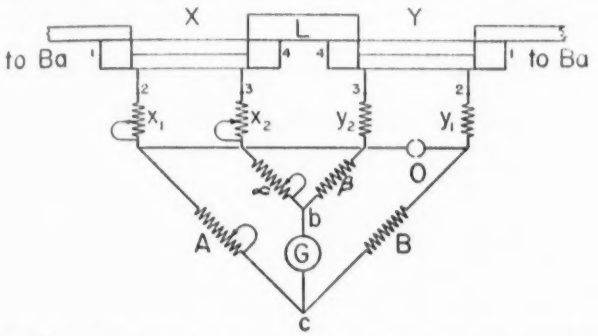

FIGURE 16.—*Thomson bridge circuit so arranged that with a suitable procedure in establishing a balance, the ratio of the resistance of A to the resistance of B equals the ratio of the direct resistance of X to the direct resistance of Y*

It should be noted that when X and Y are nonlinear there are no definite branch points corresponding to a and d of figure 9.

be replaced by a battery having an electromotive force $E=-RI$. Hence a battery may be placed in each of the branches L and O, as well as in the normal battery branch of the bridge. Since the balance of the bridge is independent of the electromotive force in the battery branch, and the bridge is so adjusted that the balance is independent of the resistances in the L and O branches, the balance is independent of the electromotive force of each of the three batteries. It is convenient to consider that these three branches are closed alternately through a battery. The stage is now set for the application of Kirchhoff's reciprocal theorem [49], from which it follows that in any balanced bridge the position of the battery and galvanometer may be interchanged without disturbing the balance. Therefore, if a battery is placed in the normal position of the galvanometer and the galvanometer is connected alternately in the normal battery branch, the O branch and in the L branch, a balance will be indicated in each case. This is the Matthiessen and Hockin bridge arrangement, and according to the Matthiessen and Hockin bridge equation [114],

$$\frac{A}{B}=\frac{x_1}{y_1}=\frac{(2143)_x}{(2143)_y}=\frac{x_2+\alpha}{y_2+\beta}, \tag{59}$$

Obviously, therefore, the procedure followed in adjusting the bridge establishes a relation between the various resistances which is independent of the proportional part of the current in each of the three branches of the bridge. Furthermore, in this relation but one of the four-terminal resistances of X and but one of the four-terminal resistances of Y appear explicitly. When all terminals of X and Y normally used as potential terminals (or normally used as current terminals) are connected to ratio arms of the bridge,

$$\frac{A}{B}=\frac{\text{the direct resistance of } X}{\text{the direct resistance of } Y}. \tag{60}$$

If, for example, terminal 1 of X were connected to the A arm of the bridge, terminal 2 were used as a connection to the battery, and other connections were as shown in figure 16, the adjustment would establish the relation

$$\frac{A}{B}=\frac{\text{the diagonal resistance of } X}{\text{the direct resistance of } Y}. \tag{61}$$

The conductors, X and Y, might be connected into the bridge in such a way that the adjustment would make the ratio of A to B equal to any one of several ratios of a four-terminal resistance of one, to a four-terminal resistance of the other. Finally, it should be noted that whether the Thomson bridge method, the potentiometer method, the Matthiessen and Hockin bridge method, or the Kohlrausch differential galvanometer method is used the same value will be found for the ratio of a four-terminal resistance of X to a four-terminal resistance of Y, provided proper adjustments are made in each case and equal precisions are attained in these adjustments.

APPENDIX 2. THERMOELECTROMOTIVE FORCES

In measurements to a precision of 1 part in 1 million, the potential drops in the bridge arms resulting from the test current must be balanced in many cases to 0.01 microvolt and in exceptional cases to 0.001 microvolt. However, conditions under which the measurements must be made are such that unless careful consideration were given to the thermoelectromotive forces they would often amount to a few or even several microvolts and change rather rapidly. As pointed out above, keeping the galvanometer circuit closed and judging the balance of the bridge from the changes in the deflection of the galvanometer following reversals of the test current eliminate the effect of the thermoelectromotive forces insofar as these remain constant during time intervals corresponding to a few periods of the galvanometer. While it is very desirable that the thermoelectromotive forces be kept constant, experience shows that a thermoelectromotive force which is changing slowly and at a fairly uniform rate does not preclude accurate balances of the bridge by a skillful observer. The principal means used for steadying the thermoelectromotive forces, most of which also reduce their magnitudes, are mentioned in the following numbered paragraphs:

1. Bridge arms, terminal blocks, the rheostat used in regulating the damping of the galvanometer, and a part of the leads between the bridge and galvanometer are placed under oil which is kept at very nearly a uniform temperature throughout by vigorous circulation.
2. The galvanometer is mounted on an inside wall, where it is fairly free from erratic changes in temperature.
3. A tight metal case completely surrounds the galvanometer, except that there is a glass window of sufficient size to permit of a view of the coil and a reflection of a light beam from the mirror.
4. The coil, coil terminals, suspensions, and terminals of the galvanometer, and the leads between galvanometer and connections in the oil bath are of copper, but the necessary connections between them are soldered.
5. That part of the galvanometer circuit outside the oil bath is thermoelectrically balanced insofar as this is practicable.
6. Most of the soldered connections outside the oil bath are thermally shunted, at least to some extent.
7. Thermal resistances are made high in parts of the circuit in which this is an advantage.
8. All soldered connections are protected from direct exposure to the general circulation of the air of the laboratory.
9. Soldered connections outside both the galvanometer case and the oil bath are kept to a minimum number.
10. Soldered connections outside the oil bath are made with the copper parts as nearly in direct contact as is practicable over areas equal to or larger than the cross section of the conductors and with only a sufficient amount of solder to produce a good thermal contact.
11. Abrupt changes in the thermal capacity per unit length of conductor in the vicinity of possible sources of changing thermoelectromotive force are avoided to as great an extent as is practicable.
12. Masses of material having relatively large thermal capacities are placed in the vicinity of more probable sources of changing thermoelectromotive force for the purpose of reducing the rate of change of the temperature.

13. The dial switches serving to adjust balances of the bridge operate in the oil bath, so are well lubricated, and this reduces the heat developed in the contacts by the operation of the switches.

14. The resistances in series with the dial switches are high relative to the resistances of the sections shunted. Consequently, variations of the thermoelectromotive force, introduced into the *A* arm of the bridge by the dial switches, are fully one order smaller than the variations of the thermoelectromotive forces in the dial switches.

Since thermoelectric balances and thermal shunts are not generally used elsewhere, an explanation of each may be in order.

(a) THERMOELECTRIC BALANCES

If a complete electric circuit is made up in part from one kind of metal and in part from another kind of metal, there must be at least two and in any case an even number of junctions between dissimilar metals. That is, the junctions occur in pairs. When these junctions are at different temperatures usually there is a thermoelectromotive force in the circuit. However, it is not necessary that all junctions be at the same temperature to avoid a thermoelectromotive force. The thermoelectromotive force would be zero if the two junctions constituting one pair were at one temperature, the two junctions constituting another pair were at another temperature, etc. This is the end sought in making thermoelectric balances. In the simplest case a thermoelectric balance consists in making the two junctions of a pair of similar construction and of placing these two junctions in such relative positions that both will be exposed to very nearly the same ambient temperature. More generally, a thermoelectric balance consists of constructions and arrangements of parts such that some type of symmetry both from the thermal and from the thermoelectric points of view is realized for two or more of several junctions in an electric circuit. For a circuit to be thermoelectrically balanced, each junction between dissimilar metals must be taken into consideration.

For the purpose of illustrating a method of realizing a thermoelectric balance, consider a soldered connection between two copper wires of the same size which are exposed to air currents of varying temperature. The connection is made similar in the two directions parallel to the circuit. The conductor is supported from opposite walls of the laboratory, and the connection is placed at some distance from either support. Consequently, air currents heat and cool the wires on opposite sides of the connection at very nearly the same rate, and keep them at very nearly the same temperature. Therefore, the two junctions, solder to copper and copper to solder, always assume very nearly the same temperature, so both the magnitude and the rate of change of the thermoelectromotive force are very small. In this case, one of the junctions is balanced against the other.

For the purpose of illustrating another method of realizing a thermoelectric balance, consider two sections of a circuit in each of which there is a soldered connection between a copper wire 1 mm in diameter, and a copper wire 0.5 mm in diameter. The two soldered connections are made similar and so located with respect to each other that in case of a changing temperature, the rate of heat transfer from a larger to a smaller wire is the same through one as through the other of the soldered connections. As a consequence, the difference in temperature of two junctions, copper to solder and solder to copper, of one connection, is very nearly the same as that of the other connection. Therefore, the two soldered connections, each constituting a thermocouple, develop very nearly the same electromotive force. Further more the arrangement is made such that in case of a current in the circuit, this current is from the larger to the smaller wire through one of the soldered connections, and from the smaller to the larger wire through the other connection. Therefore, the electromotive force developed in one of the soldered connections is in opposition to that developed in the other. In this case, one of the thermocouples is balanced against the other.

(b) THERMAL SHUNTS

A thermal shunt consists of an electrically insulated or insulating heat conductor which serves to equalize the temperature of two sections of a circuit between which there are possible sources of thermoelectromotive force. As an illustration, consider a soldered connection between a relatively large and a relatively small insulated copper wire that is exposed to air currents of varying temperature. Only sufficient insulation is removed from the ends of the wires to permit of soldering, after which the smaller wire is wrapped a few times around

the larger wire, and this portion thoroughly impregnated with shellac. Then, although air currents may heat and cool the smaller wire more rapidly than the larger, the heat transfer between the two is mainly through the electrical insulation. The result is a material reduction in the temperature difference and rate of change of the temperature difference between the two junctions, copper to solder and solder to copper, and consequently, a material reduction in the thermoelectromotive force and its rate of change.

Thermoelectric balances and thermal shunts constitute two of the simpler and more generally applicable of the effective means of reducing troubles from thermoelectromotive forces. Furthermore, each may be made to supplement the other and sundry means employed for this purpose, such as a thermal shield the circulation of air in an enclosed space, the blocking of air currents, etc.

With the arrangement used and procedure followed in establishing balances of the bridge, thermoelectromotive forces seldom constitute a limiting factor on the precision of measurement, even when there is forced circulation of the air from the general heating system at a temperature 10°C above the temperature of the bridge and 15°C above the general temperature of the laboratory or from the refrigerating system at a temperature 15°C below the temperature of the bridge and laboratory. Only in exceptional cases is it necessary to place temporarily an obstruction to the normal circulation of the air in the vicinity of the galvanometer, the terminals of standard resistors, or the connectors extending above the surface of the oil.

APPENDIX 3. INSULATION

The maintenance of sufficiently good insulation constitutes a troublesome factor, especially in the measurement of higher resistances. Referring to figure 1, it will be seen that the battery and galvanometer leads are at different potentials. Consequently, at least a small part of the current supplied by the battery passes through the insulation between them, and as a result each arm of the bridge is shunted by a resistance of the order of the resistance of the insulation between a battery lead and a galvanometer lead. In case the resistance of one arm of the bridge is 10,000 ohms, the resistance of the insulation between the battery lead and the galvanometer lead connected to it must be in excess of the 10^{10} ohms, or be compensated in some way, such for example, as by the use of a substitution method, if the error from this source is to be less than 1 part in a million. Resistances of the insulation between the battery and galvanometer leads in excess of 10^{10} ohms are not readily realized, and having been realized, it is not safe to assume that they are being maintained unless certain precautions are taken, and then only in case occasional checks justify the assumption. Usually it is assumed that a Wagner branch [96, 97] if properly designed and properly used, constitutes an effective means of avoiding errors which otherwise might result from slightly defective insulation. Originally, the bridge was equipped with a Wagner branch, but it was removed for reasons, the more important of which will be discussed later under the heading "Wagner branches." Since the removal of the Wagner branch, efforts have been made to maintain the resistance of the insulation between the battery branch and the galvanometer branch sufficiently high that the leakages between these branches do not constitute a limiting factor in the precision of measurement. However, if the insulation were to be sufficiently good to obviate errors which might arise as a result of passage of leakage current from direct current power circuits through the bridge, its resistances would have to be two or possibly three orders higher. While errors from this source are eliminated for the most part by the reversal of the connections to the battery used with the bridge, and this is the regular practice followed in balancing the bridge, a grounded metal guard is placed under all mechanical supports of the bridge, the battery and its leads, and the galvanometer and its leads. This is a very effective means of isolating the bridge from power circuits. This guard also serves in testing the insulation of battery and galvanometer leads. Furthermore, the power used in regulating the temperature and driving the stirring motor is supplied from an alternating current circuit, one side of which is grounded.

The means employed for securing good insulation are as follows: (1) The air of the laboratory is dried by refrigeration when this is necessary to keep the dew point well below the temperature of the oil bath and laboratory, not only while the measurements are being made but during a sufficient time in advance to thoroughly dry all insulating material exposed to the air. Incidentally, drying the air of the laboratory reduces the moisture content of the oil, and this helps in the maintenance of its insulating property. (2) Care is taken never to reduce

the temperature of the oil below the dew point of the air of the laboratory, as this would result in the condensation of water in the oil. (3) The oil is selected on the basis of original freedom from acids and tendency to develop acids, is protected from strong light, is kept covered when the apparatus is not in use, and is renewed at intervals of about 2 years. (4) Most of the insulating material is protected from strong light, especially when not in use. (5) Care is taken not to permit the accumulation of excessive amounts of dust or hygroscopic sulfur compounds on the surfaces of the insulators of the hard-rubber type. (6) The bridge proper and the rheostat used in regulating the damping of the galvanometer are kept under oil. (7) The galvanometer leads have but few supports and these are of amber. (8) The galvanometer case is supported on amber posts, and the galvanometer circuit is insulated from the case. (9) The battery is supported on blocks of paraffin, and the battery leads are well insulated.

However, the resistance of the insulation of the battery and its leads, the rheostats for regulating the test current, the ammeter-voltmeter, keys, etc., generally is from two to four orders lower than the resistance of the insulation of the galvanometer and galvanometer leads. It should be noted that it is not necessary that the insulation of both the battery branch and the galvanometer branch be exceptionally good. Nevertheless, when the humidity is very low, the insulation of the entire bridge circuit is so good that there may be trouble from electrostatic actions. To avoid electrostatic effects arising from the stirring of the oil, the circulating propeller is electrically connected to the guard, and the belt driving the circulating propeller is located beneath the metal tank containing the oil bath. The more serious of the remaining sources of electrostatic effects arise from movements of the observer. In case these movements cause trouble, the guard is temporarily extended under the observer so he becomes a part of it, the observer is thoroughly insulated from the floor, or a galvanometer lead is connected to the galvanometer case. Should slight defects in the insulation be suspected, in any case, their presence or absence may be confirmed easily. If present, but not very serious, usually it is a simple matter to carry out the measurement in such manner as to avoid an error from this cause.

(a) THEORY IN CASE OF DEFECTIVE INSULATION

The correction for the effect of slightly defective insulation will be designated as f. Then, following the notation used in the paper, the Wheatstone bridge equation may be written

$$X=\frac{Y}{B}A_n[1+a-a_n+f]. \tag{62}$$

In cases in which the resistances of the battery leads and galvanometer leads are negligibly small in comparison with the resistances of the bridge arms, as is usually assumed in discussions of this subject, figure 17 represents the bridge circuit with sufficient exactness for the purpose at hand. Here a, b, c, and d represent branch points of the bridge; e represents the guard extending under all mechanical supports of the apparatus; and m, n, o, and p represent the resistances of the insulation between leads, or branch points of the bridge and the guard. For this somewhat ideal arrangement, if the bridge is balanced, the relation between A, B, X, Y, m, n, o, and p is

$$\frac{X(1+Ymp/F)}{Y(1+Xop/F)}=\frac{A(1+Bmn/F)}{B(1+Ano/F)}, \tag{63}$$

where

$$F=mno+nop+opm+pmn. \tag{64}$$

Equation 63 is readily obtained by the use of Rosens [77] star-polygon transformation. If put in the form of eq 62, and if F is sufficiently large relative to Ypm, Xop, Bnm, and Ano, that terms divided by the second and higher power of F may be neglected,

$$X=\frac{Y}{B}A_n[1+a-a_n+(Xop+Bmn-Ano-Ymp)/F]. \tag{65}$$

Therefore, to a first order approximation,

$$f=\frac{Xop+Bmn-Ano-Ymp}{mno+nop+mop+mnp}. \tag{66}$$

Equation 66 shows the effect of slight defects in the insulation and the relation between these defects necessary for them to compensate each other. However, it is of little use in the solution of the particular problem at hand, since, in general, m, n, o, and p will not be known. Furthermore, it is not sufficiently general to be applicable in all cases.

Frequently a resistance is placed in one or both battery leads to reduce the potential drop across the bridge to a value less than the electromotive force of the battery and in one or both galvanometer leads or in parallel with the galvanometer

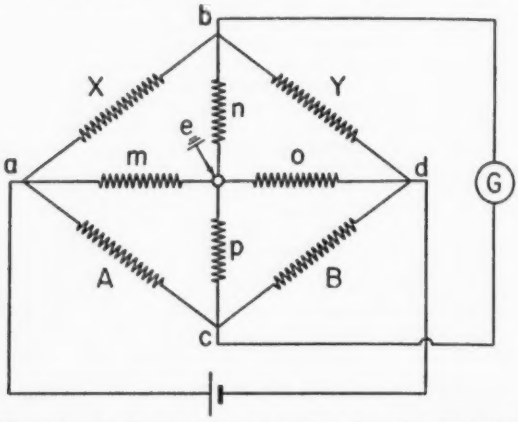

FIGURE 17.—*Wheatstone bridge circuit arranged to show the effect of slightly defective insulation of the battery leads and of the galvanometer leads.*

to adjust the damping of the galvanometer. Also major parts of the leakage through or over insulating material may be distributed near the battery and galvanometer. When this is the case, resistances in the leads make slight defects in the insulation more serious.

(b). PROCEDURE IN CASE OF DEFECTIVE INSULATION

For such a bridge circuit as shown in fig. 18 or the simpler one (shown in fig. 17), to a first-order approximation, the correction term, f, of eq 62 may be considered to be the product of two factors.[5] One of these, which will be designated J, comprises two or more resistance ratios involving only the resistances of bridge arms, galvanometer leads, and leakage paths between galvanometer leads and the guard. The other, which will be designated K, is the potential drop from the guard to the branch point b (or branch point c) divided by the potential drop from a to d.

Following the notation used above and replacing f by the product of J and K,

$$X=Y\frac{A_n}{B}[1+a-a_n+JK]. \tag{67}$$

With branch point a connected to the guard

$$K_a=X/(X+Y), \tag{68}$$

so

$$X=Y\frac{A_n}{B}[1+a_a-a_n+JX/(X+Y)]. \tag{69}$$

With branch point d connected to the guard,

$$K_d=-Y/(X+Y), \tag{70}$$

so

$$X=Y\frac{A_n}{B}[1+a_d-a_n-JY/(X+Y)]. \tag{71}$$

[5] The basis of the discussion given here will be explained in the appendix on methods of analysis, first expedient.

From eq 69 and 71 it follows that

$$J = a_d - a_a. \tag{72}$$

From eq 69 and 72, or 71 and 72, it follows that

$$X = Y\frac{A_n}{B}\,[1 + (a_a Y + a_d X)/(X + Y) - a_n]. \tag{73}$$

In like manner it may be shown that if balances of the bridge are made with branch points b and c alternately connected to the guard

$$X = Y\frac{A_n}{B}\,[1 + (a_b A + a_c X)/(A + X) - a_n]. \tag{74}$$

Referring to eq 73 or eq 74, it will be seen that by making two balances of the bridge instead of the usual one, the effect of slight defects in the insulation of battery and galvanometer leads is taken into account. For eq 73 or 74 to give results accurate to 1 part in 1 million, $a_a - a_d$ or $a_b - a_c$ must lie in the range 0.001 to −0.001. However, these two equations are independent of each other, so usually one or the other will be applicable.

It should be pointed out that the situation is somewhat more complicated than would appear from the above discussion or an inspection of figure 18. For reasons which are stated on page 239, the balance of the bridge is judged from changes in the deflection of the galvanometer following reversals of connections to the leads of the battery. These reversals are made at points at some distances from the battery terminals. As a consequence, the direction of the potential gradient in the insulation may be reversed in some and not in others of the leakage paths, so the correction term may be different in the two cases. Consequently, when eq 73 or 74 is used an appropriate procedure should be followed in establishing the extra balances of the bridge.

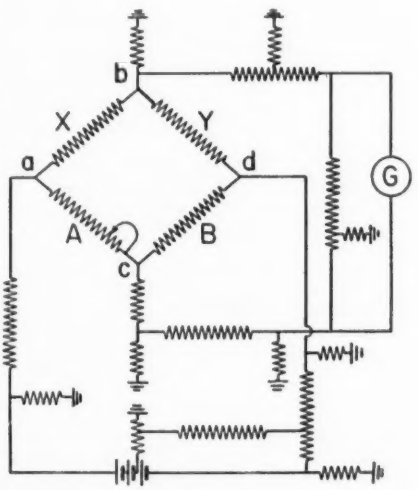

FIGURE 18.—*Wheatstone bridge circuit with leakage paths between battery leads and the guard, and between the galvanometer leads and the guard.*

(c) TEST OF INSULATION

However, what is desired is not an evaluation of a correction term but an assurance that it is so small that it may be neglected without reducing the precision of the measurements. Keeping in mind the relations given above, it is a simple matter to more or less definitely locate defects in the insulation if these defects are really serious, although it may or may not be a simple matter to remedy the difficulty. Since the insulation of the galvanometer, galvanometer leads and rheostats used in regulating the damping of the galvanometers is generally much better than the insulation of the battery, battery leads, switches, and rheostats used in adjusting the current through the bridge, the insulation may be tested by making K of eq 67 have first a positive and then a negative value somewhat larger than the value it can possibly have as the bridge is used, and noting the resulting changes, if any, in the balances of the bridge.

The value K naturally assumes presumably will never lie outside the range from plus to minus the electromotive force of the battery, supplying the test current, divided by the potential drop from a to d. Usually, therefore, an appropriate

change in K may be made by connecting an auxiliary battery (having an electromotive force of about one order higher than the electromotive force of the battery supplying the test current) between the guard and branch point a (or d) and then reversing connections to this auxiliary battery. This should be done with the reversing switch in the battery branch of the bridge in one position and then repeated with the reversing switch in the other position, the battery branch being closed in both cases.

Of the locations not so far considered, at which slightly defective insulation might introduce errors, the more probable is in the auxiliary apparatus used in finding the reading of the dial switches of the A arm of the bridge for which the ratio of A to B is 10. At the time this apparatus was constructed the importance of providing a means of testing the insulation was not appreciated. For example, should the resistance of the insulation between one of the end terminal blocks and one of the central terminal blocks be as low as 10^8 ohms, the reading obtained would correspond to a ratio differing from 10 by about 2 parts in 1 million. However, checks by a number of standard resistors singly and then in series have not indicated any significant error from this source.

It should be pointed out that when nominally equal resistances are being compared, either by substitution one after another in the same arm of the bridge or by interchange between arms, errors which otherwise would result from slightly defective insulation are automatically eliminated to the extent that the shunting effects remain constant during a series of measurements. Also, that as the apparatus is used, leakages within the A and B arms of the bridge are automatically taken into account to the extent that they remain constant during a series of measurements.

In general, leakages within resistance standards have the effect of lowering their resistances, rather than limiting the precision of the measurements. In exceptional cases in which the conduction through the insulation is sufficient to result in noticeable electrolytic polarization the usual precision of measurement cannot be obtained.

It has been assumed that direct leakage from any part of the bridge circuit to points within X or Y is negligible. Also that direct leakage between battery and galvanometer leads is negligible. The bridge is kept under oil of good quality, and the battery and galvanometer leads are brought out, so that there is little possibility of a current from one to the other except through bridge arms or by way of the guard. So presumably these assumptions are justifiable.

(d) WAGNER BRANCHES

It was mentioned that originally the bridge was equipped with a Wagner branch. Usually when a Wagner branch is used with a Wheatstone or other four-arm bridge, it is attached as shown in figure 19. Here W_a and W_b represent the arms of the Wagner branch. The bridge and the Wagner branch are adjusted by successive approximations. First, the bridge is adjusted with the key k_w open, so that there is no current in the galvanometer. Second, with the galvanometer connected between branch point b (or c) and the guard or with the galvanometer in its normal position, and the key, k_w, closed, the Wagner branch is so adjusted that there is no current through the galvanometer. This second adjustment, if it involves any marked changes in the ratio of W_a to W_b, disturbs the first adjustment to some extent. Therefore, the first adjustment is repeated and this disturbs the second adjustment slightly, etc. However, successive adjustments require smaller and smaller changes, and after the changes become insignificant, the galvanometer branch is at the potential of the guard. Then there is no leakage current through the leakage path, q, or any arm of the bridge. Therefore, since there is no current through the galvanometer, the Wheatstone bridge equation gives the relation between X, Y, A, and B.

If this adjustment were valid with the key k_1 against the + battery terminal block and the key, k_2, against the − battery terminal block, the relation between the resistances would be as follows:

$$\frac{W_a(1+W_b/q)}{W_b}=\frac{X}{Y}=\frac{A}{B}. \tag{75}$$

However, with this adjustment the galvanometer branch will not be at the potential of the guard with both keys k_1 and k_2 against + battery terminal blocks, as shown in figure 18, or with k_1 against a −, and k_2 against a + battery terminal

block. More specifically, when a switch, such as is shown in figure 19, is used in reversing the current through the bridge and this switch is located between the bridge and a leakage path between a battery lead and the guard, no adjustment can be made such that the galvanometer branch will be at the potential of the guard for all permutations of the settings of the keys k_1, k_2, and k_w. If then there is a leakage path from a galvanometer lead to the guard, no adjustment can be made such that there is no current through the galvanometer with the current through the bridge alternately in one and then in the other direction, when the key k_w is open and also when the key k_w is closed.

Notwithstanding this difficulty it is possible, in some cases only, to make an adjustment such that the relation between X, Y, A, and B is that given by the

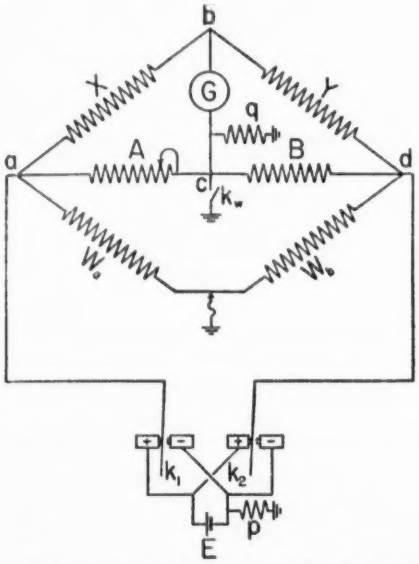

FIGURE 19.—*A Wagner branch connected in parallel with the battery branch of a Wheatstone bridge.*

W_a and W_b represent the arms of the Wagner branch, and p and q represent leakage paths between the bridge circuit and the guard.

Wheatstone bridge equation. Adjustments of the bridge and of the Wagner branch are made by successive approximations, judging balances by changes in the deflection of the galvanometer following reversals of the test current. In adjusting the bridge, the key k_w is left open, while in adjusting the Wagner branch, it is kept closed. When proper adjustments have been realized there may be a fairly large current through the galvanometer with the key k_w open and a different fairly large current through the galvanometer with the key k_w closed. However, the magnitudes and directions of these currents are independent of the direction of the current through the bridge.

The situation is not nearly so complicated if the Wagner branch is attached to branch points b and c of the bridge, as is shown in figure 20. With this arrangement adjustment by successive approximations leads to a condition such that there is no current through the galvanometer with any possible combination of the settings of the keys k_1, k_2, k_w, and k_w. While this adjustment does not bring the galvanometer to the potential of the guard or prevent a leakage between a galvanometer lead and the guard, it nevertheless leads to the relation between X, Y, A, and B given by the Wheatstone bridge equation.

This discussion of the Wagner branch is by no means comprehensive. It should however serve to show that an attachment of a Wagner branch to the NBS precision bridge would not constitute a simple and convenient means of obviating the need of good insulation.

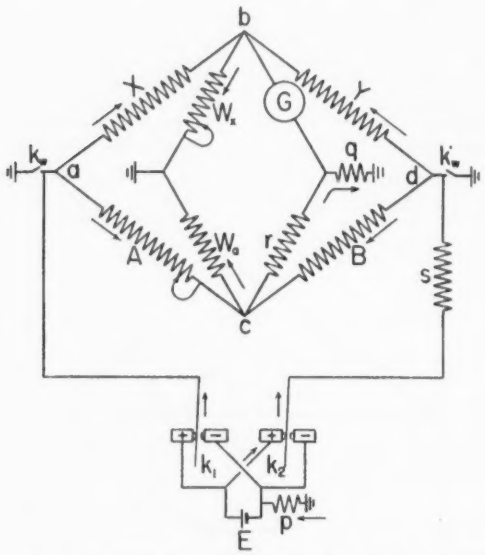

FIGURE 20.—*A Wagner branch connected in parallel with the galvanometer branch of a Wheatstone bridge*

Here the arrows indicate the direction of the leakage current in each branch of the network when the potential of all bridge arms is higher than the potential of the guard.

APPENDIX 4. OPTICAL SYSTEMS OF GALVANOMETERS

While balances of the bridge are being established, the deflection of the galvanometer is ever changing, so it requires the concentrated attention of the observer. Watching the deflection and separating those componeuts of the deflection resulting from a lack of balance of the bridge from those resulting from other causes constitute the most fatiguing part of a series of measurements. As fatigue is a factor affecting the precision of a series of measurements the optical system should be as suitable for the purpose as can be obtained without excessive refinements. What constitutes the best obtainable optical system depends on the skill of the available optician, personal preference, and physiological factors.

(a) DESCRIPTION OF OPTICAL SYSTEM

In the system used in the precision measurements of resistance made in this Bureau an image of a line source of light is focused on a ground-glass scale. The scale is placed at a distance of about 1.5 m in front of the galvanometer mirror and is adjustable in height so as to accommodate it to different observers while standing. The graduations and ground surface are on the observer's side of the scale, which is a matter of very considerable importance, since it serves to almost completely eliminate direct reflections from the scale.

The light source is a 35-w straight filament lamp with a cylindrical bulb. However, it is operated far below normal intensity for reasons which will be explained presently. The lamp is enclosed in a cylindrical metal case with openings at the top and bottom for ventilation and an opening on the side which can be so adjusted as to exclude from the light beam reflected from the galvanometer mirror, images resulting from reflections within the lamp bulb. The galvanometer mirror is 1 cm in diameter and about 0.5 mm in thickness. The quality of the mirror and galvanometer window is such that practically theoretical resolving power is realized.

All direct illumination back of the ground-glass scale is avoided. Consequently, the only marked contrasts arise from the graduations and the image of the light source. The grinding is rather fine but not so fine but that both the graduations and the image are seen distinctly when viewed at an angle of as much as 20° with respect to the normal to the scale. This permits the use of

both eyes and does not require that the observer keep his head in a very definitely fixed position. The graduations are fine, distinct, black lines with 1-mm spacing.

The lamp and scale are supported from a side wall and the ceiling, both of which are masonry. Also the galvanometer is supported from a side wall. Consequently, mechanical vibrations of the scale and image are seldom troublesome.

With the lamp operated at normal intensity, the image appears rather wide and excessively bright. Therefore, the power supplied to the lamp is reduced somewhat below the point at which, on dull days and with the window shades open, the diffraction fringes do not contribute to the apparent width of the image and a suitable contrast is obtained. As the outside illumination changes the intensity of the light source is not changed but occasionally one or more of the window shades are closed or the lights of the laboratory turned on. The general illumination is so adjusted that the contrast between the graduations and ground glass is approximately the same as the contrast between the ground glass and image.

In cases in which the combined sensitivity of the bridge and galvanometer are barely sufficient for obtaining the desired precision, repetitions of series of measurements show that an exceptionally skillful observer can so nearly balance the bridge that that component of the electromotive force resulting from the lack of balance of the bridge does not exceed 0.001 microvolt. Since the sensitivity of the galvanometer then would be 30 mm/microvolt, it follows that 0.001 microvolt corresponds to changes in the deflection of 0.06 mm, following reversals of the direction of the test current. To associate a change in the deflection that is only slightly larger than this with a change in the direction of the test current (while the deflection is changing gradually as a result of one or more of several causes, abruptly on reversals of the test current because of a lack of a perfect inductive balance, and erratically as a result of one or more of several possible causes); requires that the deflection be observed continuously while the test current is reversed several times, at intervals corresponding approximately to the period of the galvanometer. However, a balance of the bridge to 0.001 microvolt can be made only when the thermoelectromotive forces are remaining reasonably constant, the inductances of the bridge are reasonably well balanced, local and microseismic vibrations are not troublesome, electromotive forces induced by changes of the magnetic field in the laboratory are not troublesome, and conditions otherwise are favorable.

(*b*) QUALITY OF OPTICAL SYSTEMS AND SENSITIVITY OF BRIDGES

Since the optical system of the galvanometer is the last of the factors affecting the sensitivity of bridges to be discussed at this time, it is in order to give an illustration of how a consideration of most of these factors leads to a fairly definite conclusion. For this purpose let the reader assume that he has the problem of comparing a number of 10-ohm standard resistors, that he has at hand a bridge suitable for the purpose, and that he has prepared a specification for a galvanometer, but before purchasing a galvanometer wishes to know if the specification is adequate and satisfactory for the realization of a precision of 1 part in 1 million in the comparisons.

Let it be assumed that:

1. The period of the galvanometer is to be 7 seconds.
2. The galvanometer is to be critically damped with an external resistance of 60 ohms.
3. The sensitivity of the galvanometer is to be such that with 1 microvolt, in the circuit giving critical damping, the deflection is five scale divisions, with the distance between the mirror and scale that which is to be used.
4. The optical system of the galvanometer is to be of such quality that a change of the deflection of 0.1 scale division is readily detectable.
5. It is estimated that local disturbances will cause random change in the apparent equilibrium position of the galvanometer of not more than 0.1 scale division during time intervals of 7 seconds.
6. The resistance between the galvanometer terminals of the bridge is 60 ohms.
7. The comparisons are to be made by substituting the 10-ohm standard resistors successively in the X arm of the bridge.
8. A 10-ohm standard resistor is to be placed in the Y arm of the bridge and is not to be disturbed during the series of measurements.
9. Balances of the bridge are to be checked by a single observation of the change in the deflection of the galvanometer following a reversal of the test current, with no provision for establishing inductive balances.

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10. The divergence of the load coefficients of the series of 10-ohm standard resistors, to be compared, is 10 parts in 1 million per watt dissipation.

As previously pointed out (eq 26*x*), the change in the deflection of the galvanometer following a reversal of the test current is

$$2\,D\,E_x\,Y/(X+Y),$$

provided, as according to 2 and 6, the specified damping is realized without the use of a shunt. Here E_x, the potential drop in the X arm, is not assumed to be known but is to be determined from the data given above. Since the sensitivity of the galvanometer, D, is five scale divisions per microvolt and X and Y are equal, the change in the deflection of the galvanometer resulting from 1 part in 1 million lack of balance of the bridge is five E_x scale divisions, provided E_x is expressed in volts.

From 1, 2, 4, 5, and 9 it follows that the bridge may be out of balance nearly by an amount corresponding to a change of the deflection of the galvanometer of 0.2 scale division without being detected. Consequently, the minimum discernible

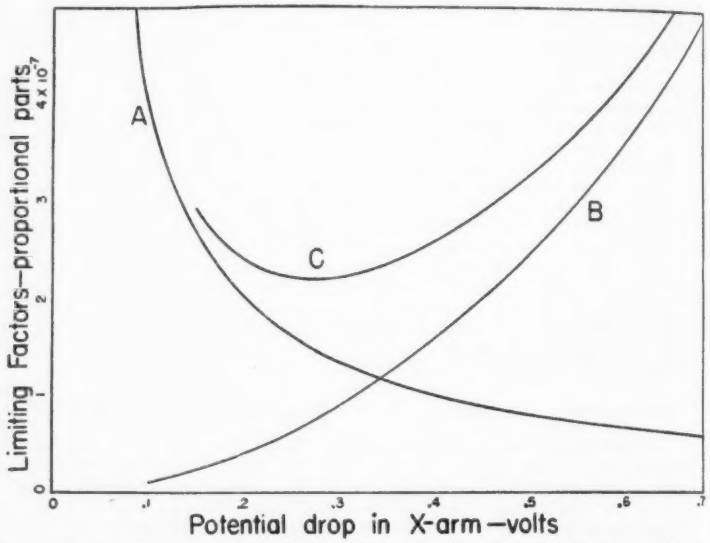

FIGURE 21.—*Sensitivity of bridge and galvanometer (an illustrative problem).*

Curve *A* shows minimum discernible lack of balance of the bridge; curve *B*, effect of heating; and curve *C*, sum of factors limiting the sensitivity.

lack of balance of the bridge may be taken as 1 in 25 million E_x. Plotting the minimum discernible lack of balance of the bridge as ordinates and E_x as abscissas gives curve *A* of figure 21.

From 7 and 10, since the resistance in the X arms is 10 ohms, it follows that for the series of measurements the divergence of the effects of heating is $E_x^2 \times 10^{-4}$. Plotting E_x^2 as ordinates and E_x as abscissas gives curve *B*. Adding the ordinates of curves *A* and *B* gives curve *C*, the sum of the various factors limiting the sensitivity. It will be observed that the maximum of what may be called the effective sensitivity is 2.2 in 10 million, or about 1 in 4½ million, and that this sensitivity is obtained with a potential drop of approximately 0.27 volt in the X arm of the bridge. Obviously this sensitivity is somewhat more than that required for a precision of 1 part in 1 million.

In this analysis it has been assumed that the combined effect of the various factors limiting the effective sensitivity is obtained by the addition of their individual effects, that is, the analysis is based on the worst, not the probable, of the assumed conditions. It may be concluded, therefore, that a galvanometer having the operating characteristics specified would be suitable for obtaining the desired precision in the series of measurements, and inasmuch as there would be no difficulty in obtaining a galvanometer having these operating constants, it may be concluded that the specification is satisfactory.

APPENDIX 5. METHODS OF ANALYSES (THREE EXPEDIENTS)

Instead of Kirchhoff's laws or Maxwell's "mesh currents" simpler expedients have been used in this paper in the solution of network problems. For the most part these are so obvious or so well known that no explanation need be given, or reference is made to publications in which they are discussed. However, three of the expedients used deserve special mention. The first of these consists in the substitution of a battery in place of a resistance in which there is a current, the second consists in the substitution of a very high resistance and battery of very high electromotive force in place of a low (or zero) resistance and battery of low electromotive force, while the third consists in a separation and synthesis of the effects of the independent resistances of a nonlinear four-terminal conductor. This is accomplished by the substitution of first four, then five, and finally another five linear conductors in place of the nonlinear four-terminal conductor. These substitutions present no experimental difficulties, since they are made not in an actual circuit but in a diagram representing the circuit or merely in a mental picture.

(a) CLASSICAL FORM OF SOLUTION OF UNBALANCED WHEATSTONE BRIDGE

Before considering any one of these three expedients a few of the many possible expressions for the electromotive force or current in the galvanometer circuit of an unbalanced Wheatstone bridge will be stated, for the purpose of comparison with corresponding expressions, one of which will be derived by the use of the first and another by the use of the second of these expedients.

The expression given by Maxwell and others, and which will be referred to as the "classical solution," is

$$I_g=\frac{E_b B\Delta X}{RG(A+B+X+\Delta X+Y)+R(A+X+\Delta X)(B+Y)+ \\ +G(A+B)(X+\Delta X+Y)+XY(A+B)+AB(X+\Delta X+Y)+\Delta XY(A+B).} \tag{76}$$

Here I_g is the current in the galvanometer branch and G is the resistance of the galvanometer branch. The significance of the other characters are as explained in connection with figure 1.

It has been shown in this paper (eq 6) that

$$E_g=\frac{EX}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right], \tag{77}$$

from which it follows [30, 71, 88, 108] that

$$I_g=\frac{EX}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right]/R_g, \tag{78}$$

where R_g is the resistance to an electromotive force in the galvanometer branch, that is, the resistance of the galvanometer branch plus the resistance of the bridge between its galvanometer terminals with the galvanometer branch open and the battery branch closed.

(b) FIRST EXPEDIENT (HYPOTHETICAL BATTERY OF ZERO RESISTANCE)

For a battery having no resistance to be equivalent to a resistance in which there is a current, the electromotive force of the battery must be equal to minus the product of the current and resistance. As simple illustrations of the application of this expedient, consider the circuit shown in figure 1. Here the current distribution in the bridge arms and galvanometer branch may be considered to depend on the potential drop from a to d, regardless of how this potential drop is produced. We are therefore at liberty to consider that the potential drop is produced in any way which suits our convenience. One of the ways in which the potential drop might be produced would be by replacing the resistance, R, in the battery branch in which there is a current, I, by a second battery having no resistance and an electromotive force equal to $-RI$, or by replacing the entire battery branch by a battery having no resistance and an electromotive force $E_1=E_b-RI$. If this were done, the potential drop from a to d would not change on opening the galvanometer branch. The potential drop across the break in the galvanometer branch is given by eq 6, if E_1 is substituted for E. Another simplification accomplished by this expedient is that from the standpoint of the

resistance to an electromotive force in the galvanometer branch, branch points *a* and *d* are connected by a conductor of zero resistance. Consequently, an expression for the resistance to an electromotive force in the galvanometer branch can be written from an inspection of figure 1, and is

$$G+\frac{AB}{A+B}+\frac{(X+\Delta X)\,Y}{X+\Delta X+Y}.$$

Consequently,

$$I_g=\frac{E_1X}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right]\Big/\left(G+\frac{AB}{A+B}+\frac{(X+\Delta X)\,Y}{X+\Delta X+Y}\right). \tag{79}$$

This equation or its equivalent was communicated to me orally by Leo Behr, without an analytical proof. However, his original analysis was the same as that given here.

This expedient, in some cases, is helpful in the solution of complicated network problems as well as simpler problems, such as that just considered. As an illustration, consider the bridge circuit shown in figure 18 and consider that the bridge is balanced. Hypothetically, the battery, battery leads, and leakage paths between the guard and battery, and guard and battery leads may be replaced by two or more batteries (each having the appropriate electromotive force) connected as shown in figure 22, or otherwise, without changing the current or potential relative to the guard at any point in the bridge arms, galvanometer leads, and leakage paths between the guard and galvanometer leads.

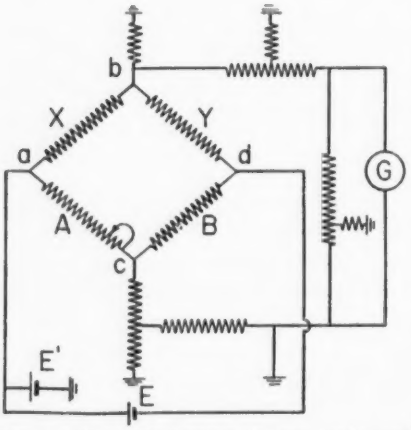

FIGURE 22.—*The battery, battery leads, and leakage paths (between battery leads and the guard only) shown in figure 18 replaced by two batteries.*

Obviously, if $E'=EX/(X+Y)$, no current is drawn from the battery whose electromotive force is E' and the usual Wheatstone bridge equation gives the relation between X, Y, A, and B. However, in general the relation between X, Y, A, and B will depart from the Wheatstone bridge relation proportionally to $(E'/E)-(X/(X+Y))$, while the proportionality factor depends upon the relative resistances of the bridge arms, leakage paths from the galvanometer leads to the guard, and sections of the galvanometer leads. By deliberately changing $(E'/E)-(X/(X+Y))$ first to one and then to another known value and noting the resulting changes in the balance of the bridge, both the initial value of $(E'/E)-(X/(X+Y))$ and the proportionality factor may be found. This is the basis of the discussion given in the appendix on insulation.

(c) SECOND EXPEDIENT (HYPOTHETICAL BATTERY OF HIGH ELECTROMOTIVE FORCE AND HIGH RESISTANCE)

For a high resistance and battery of high electromotive force to be equivalent to a low (or zero) resistance and battery of low electromotive force, it is necessary that the high electromotive force minus the product of the high resistance and current be equal to the low electromotive force minus the product of the low resistance and current, and that the current be the same in the two cases. As a simple illustration of the application of this expedient, consider again the circuit shown in figure 1. Here the current distribution in the bridge arms and galvanometer branch may be considered as dependent on the current in the battery branch regardless of how this current is produced. We are therefore at liberty to consider that the current is produced in any way which suits our convenience.

One of the ways in which the current I in the battery branch might be produced would be by increasing the resistance R indefinitely and increasing the

electromotive force E_b sufficiently to give this current. If this were done the galvanometer branch could be opened without changing the current I in the battery branch. Furthermore, from the standpoint of an electromotive force in the galvanometer branch, the resistance of the battery branch would be infinite. Consequently, an expression for the resistance to an electromotive force in the galvanometer branch could be written from an inspection of figure 1. With the galvanometer branch open it is readily seen that the current in the X arm is $I(A+B)/(X+\Delta X+Y+A+B)$ and in the A arm is $I(X+\Delta X+Y)/(X+\Delta X+Y+A+B)$. These currents multiplied by the resistances $(X+\Delta X)$ and A give the potential drop from a to b and from a to c, while the difference between these two potential drops is the electromotive force appearing in the galvanometer circuit when the galvanometer branch is closed. With the galvanometer branch closed it is readily seen that the resistance to an electromotive force in the galvanometer branch is $G+(X+\Delta X+A)(Y+B)/(X+\Delta X+Y+A+B)$. Consequently, the current in the galvanometer branch

$$I_g=\frac{I[(X+\Delta X)(A+B)-A(X+\Delta X+Y)]/(X+\Delta X+Y+A+B)}{G+(X+\Delta X+A)(Y+B)/(X+\Delta X+Y+A+B)}, \tag{80}$$

which reduces to

$$I_g=\frac{IB\Delta X}{G(A+B+X+\Delta X+Y)+(A+X+\Delta X)(B+Y)}. \tag{81}$$

This is an equation given without proof by Jaeger [39].

Equations 76, 77, 78, 79, and 81, are exact, and two or more of them might be applied to the same bridge at the same time. That is, each of these equations is a different form of the solution of the same problem. To reduce them to a common form, eq 76 for example, would require a determination of the relation between E, E_1, I, and E_b, and of an expression for R_g in terms of G, $X+\Delta X$, Y, A, B, and R. A comparison of the derivation of eq 76 either by the method used by Heaviside or by the method used by Maxwell with that used in deriving eq 79 and that used in deriving eq 81 would serve to show the relative simplicity of the methods used here.

(d) THIRD EXPEDIENT (SEPARATION AND SYNTHESIS OF THE EFFECTS OF THE SIX INDEPENDENT RESISTANCES OF A FOUR-TERMINAL CONDUCTOR)

Before considering the separation and synthesis of the effects of the independent resistances of a nonlinear four-terminal conductor, it is important to know the number and nature of these resistances and the purpose of the separation and synthesis. In the appendix on terminals and contacts it was pointed out that a four-terminal conductor has but two independent four-terminal resistances. These are the direct resistance and the cross resistance, which here will be designated D and C. The remaining independent resistances are the terminal resistances, which are of the three-terminal type. These are

$$\left.\begin{aligned} R_1&=(1124)=(1142)=(4211)=(2411)\\ R_2&=(2231)=(2213)=(1322)=(3122)\\ R_3&=(3342)=(3324)=(2433)=(4233)\ \text{and}\\ R_4&=(4431)=(4413)=(1344)=(3144). \end{aligned}\right\} \tag{82}$$

(See appendix on terminals and contacts for notation)

The purpose of the separation and synthesis of the effects of these six resistances is to obtain analytical solutions of complicated networks composed of linear conductors and nonlinear four-terminal conductors. Normally, little or nothing would be gained by separating the effect either individually or collectively of the terminal resistances of the four-terminal conductors from the effect of the resistances of the linear conductors connected to the terminals. Therefore the only separations which will be considered are (a) of the effect of the direct resistances from the combined effect of the cross resistances, the terminal resistances, and the resistances of the linear conductors; and (b) of the effect of the cross resistances from the combined effect of the direct resistances, the terminal resistances, and the resistances of the linear conductors. Since the solution must contain the combined effect of the terminal resistances and the resistances of linear conductors, in general it is necessary to determine three effects. These three effects may be determined by considering (1) that each nonlinear four-terminal conductor, constituting a part of a network, has four linear resistances arranged as shown in figure 23 (a), (2) that each nonlinear four-terminal conductor

has five linear resistances arranged as shown in figure 23 (b), and (3) that each nonlinear four-terminal conductor has five linear resistances arranged as shown in figure 23 (c).

When in the analysis of a network problem any of the well-known procedures is followed in the treatment of the linear conductors, the first consideration gives a partial solution which is complete, except that it contains neither the effect of the direct resistances nor the effect of the cross resistances of the nonlinear four-terminal conductors. The second consideration gives a partial solution which is complete, except that it does not contain the effect of the cross resistances of the nonlinear four-terminal conductors. The third consideration gives a partial solution which is complete, except that it does not contain the effect of the direct resistances of the nonlinear four-terminal conductors.

The second partial solution minus the first partial solution gives the effect of the direct resistances, that is, the separation (a); while the third partial solution minus the first partial solution gives the effect of the cross resistances, that is, the separation (b). The addition of the effect of the direct resistances and the effect of the cross resistances to the combined effect of the terminal resistances and resistances of linear conductors constitutes a synthesis of these three effects, and consequently gives a complete solution of the network problem. Obviously, the complete solution may be obtained by the addition of the second and third, and

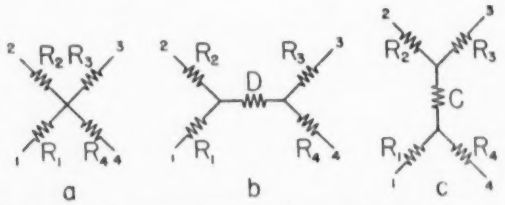

FIGURE 23.—*Hypothetical arrangements of linear resistances for separating the effect of the direct resistance and the effect of the cross resistance from the effect of the terminal resistances of a nonlinear four-terminal conductor.*

a, four linear resistances equivalent to the four terminal resistances of a nonlinear four-terminal conductor; *b*, five linear resistances equivalent to the direct resistance and the four terminal resistances of a nonlinear four-terminal conductor; *c*, five linear resistances equivalent to the cross resistance and the four terminal resistances of a nonlinear four-terminal conductor.

subtraction of the first of the partial solutions. If the second or third of these partial solutions contains the effect of the direct resistances or of the cross resistances as additive terms only, that is, if the separation (a) or the separation (b) occurs more or less automatically, the transfer of the additive terms occurring in one of these partial solutions to the other of these partial solutions gives the complete solution, so the first partial solution is not required. In cases in which the combined resistances of terminals of the nonlinear four-terminal conductors and linear conductors connected to the terminals are sufficiently high relative to the four-terminal resistances of the nonlinear four-terminal conductors that first order approximations of the effects of the four-terminal resistances are sufficient, the effect of the direct resistances can always be obtained as additive terms in the second partial solution, and the effect of the cross resistances can always be obtained as additive terms in the third partial solution.

To show how this method of analysis works out in practice, consider that three linear conductors having resistances of 0.43, 1, and 1.37 ohms respectively, are connected to a nonlinear four-terminal conductor, Q, as shown in figure 24 (a), that the six independent resistances of Q are

$$R_1 = .07 \text{ ohm},$$
$$R_2 = .13 \text{ ohm},$$
$$R_3 = .05 \text{ ohm},$$
$$R_4 = .03 \text{ ohm},$$
$$D = .08 \text{ ohm, and}$$
$$C = .03 \text{ ohm},$$

and that the problem is to find first the potential drop from M to 3 with 1 ampere in the circuit and second the resistance of that part of the circuit included between M and N. To more definitely visualize these problems, it may be assumed that Q consists of a circular piece of sheet metal to which terminal posts are soldered.

The first partial solution is obtained from the arrangement of linear conductors in figure 24 (b). For this arrangement the resistance between M and the junction R_1, R_2, R_3, and R_4 (see fig. 23, a) is $(.5\times1.5)/2$, or .3750 ohm. Therefore, since the current is 1 ampere, the potential drop from M to 3, $E_1=.3750$ volt.

The second partial solution is obtained from the arrangement of linear conductors shown in figure 24 (c). If E_2 represents the potential drop from M to 3 for

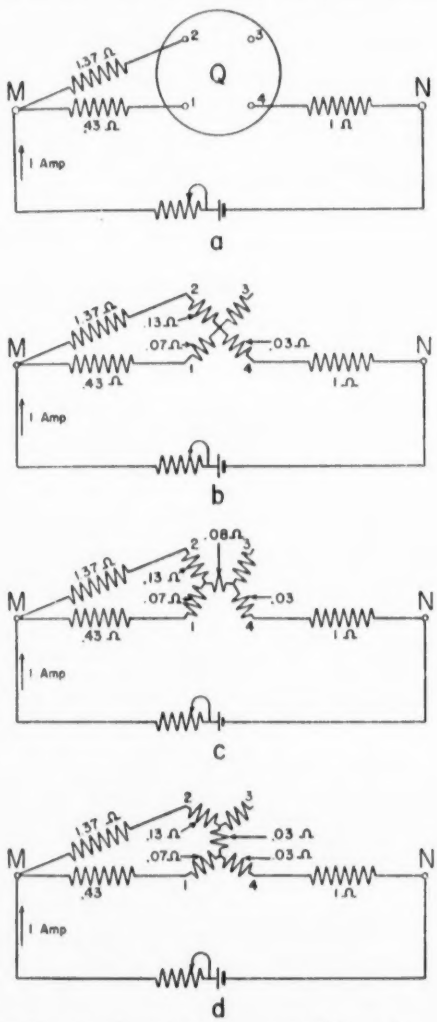

FIGURE 24.—*A simple network containing one nonlinear four-terminal conductor.*

a, a complete circuit in which *Q* represents the nonlinear four-terminal conductor; *b*, a circuit showing the terminal resistances of *Q* as linear resistances; *c*, a circuit showing the terminal resistances and the direct resistance of *Q* as linear resistances; and *d*, a circuit showing the terminal resistances and the cross resistance of *Q* as linear resistances.

this arrangement, obviously, E_2 is then larger than E_1 by 0.08 volt, so $E_2=.4550$ volt.

The third partial solution is obtained from the arrangement shown in figure 24 (d). If E_3 represents the potential drop from M to 3 for this arrangement,

it is readily seen that the current in the upper branch is $(.43+.07)/(.43+.07+.03+.13+1.37)=.2463$ ampere. This current multiplied by 1.5, the resistance of the upper branch between M, and junction of R_2, C, and R_3 (see fig. 23, c) gives $E_3=.3695$ volt. Therefore, the potential drop from M to 3 of the actual circuit, $E=E_2+E_3-E_1=.4495$ volt.

Since in the second partial solution the effect of the direct resistance of Q appeared as an additive term, namely 0.08 volt, the first partial solution might have been omitted and the complete solution obtained by transferring this additive term to the third partial solution. This would have given $E=E_3+.08=.4495$ volt.

Considering now the second problem and letting r_1, r_2 and r_3 be the resistances of that part of the circuit between M and N for the arrangements shown in figures 23 *b*, *c*, and *d*, it will be seen that the effect of the direct resistance, which is 0.08 ohm, will appear in the solution for r_2 as an additive term with a coefficient of unity. Therefore, it is not necessary to solve for r_1 or r_3.

Referring to figure 23 (d), it is readily seen that

$$r_3=\frac{(1.37+.13+.03)\ (.43+.07)}{1.37+.13+.03+.43+.07}+.03+1=1.4069 \text{ ohms.}$$

Adding to this the resistance contributed by the direct resistance of Q, namely .08 ohm, gives for the resistance of that part of the actual circuit between M and N of figure 23 (a)

$$r=1.4069+.08=1.4869 \text{ ohms.}$$

The values stated for the six independent resistances of the nonlinear four-terminal conductor, Q, may be realized by the use of eight linear conductors having the resistances and arrangement shown in figure 25. If therefore these eight linear conductors are substituted for the nonlinear four-terminal conductor, the problems just considered become problems in linear networks, and the solutions given may be checked by well-known methods.

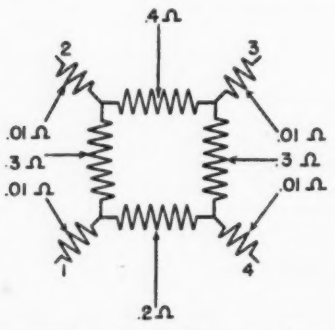

FIGURE 25.—*Eight linear conductors so arranged and having such resistances as to be equivalent to the nonlinear four-terminal conductor, Q, of figure 24 (a).*

A somewhat more complicated problem is a determination of the effects of the three different current distributions in each of the four terminal blocks involved in changing three resistance coils from a series to a parallel connection and in measuring the resistances of the paralleling connections. A solution of this problem would serve to give a better idea as to how the method of analysis under consideration works out in practice, and at the same time give an answer to a question that frequently arises in connection with precise resistance comparisons.

Referring to figure 14, the problem is to find the ratio of the four-terminal resistance of the three 150-ohm resistance sections with the paralleling connectors L_1 and L_2 removed to their four-terminal resistance with these connectors as shown, using in each case terminals 1 and 2 of terminal block *a* and terminals 1 and 2 of terminal block *d* as the four terminals. The resistances of the paralleling connections L_1 and L_2 are to be measured as four-terminal conductors. In the measurement of L_1, L_2 is removed and terminals 1 and 2 of terminal block *a* and terminals 1 and 2 of terminal block *b* are used as the four terminals. In the measurement of the resistance of L_2, L_1 is removed and terminals 1 and 2 of terminal block *c* and terminals 1 and 2 of terminal block *d* are used as the four terminals.

It will be assumed that all conductors are linear except the terminal blocks *a*, *b*, *c*, and *d*; and that each of these is a nonlinear four-terminal conductor. Aside from what is obvious, the problem resolves itself into a determination of the effects of the direct and cross resistances of the terminal blocks *a*, *b*, *c*, and *d*.

With reference to the terminal resistances of the terminal blocks, it will be seen by reference to figure 14 that R_3 of each terminal block is included in the measurement of L_1 or L_2, R_1 of terminal blocks *a* and *d* are outside the resistances

under consideration, R_1 of terminal blocks *c* and *d* may be considered as a part of a 150-ohm resistance section, and in each case R_4 may be considered as a part of a 150-ohm resistance section. Consequently, none of the terminal resistances of any of the terminal blocks need appear explicitly in the analyses of the problem. Furthermore, the effects of the direct resistances and of the cross resistances of the terminal blocks must be small relative to the effects of the 150-ohm resistance sections, so there is no need of making the first partial solution.

Considering that the terminal blocks are replaced by linear conductors representing the direct resistances of the terminal blocks gives the arrangement shown in figure 26 (a), and considering that the terminal blocks are replaced by linear conductors representing the cross resistances gives the arrangement shown in figure 26 (b). Therefore, the second partial solution may be obtained from a

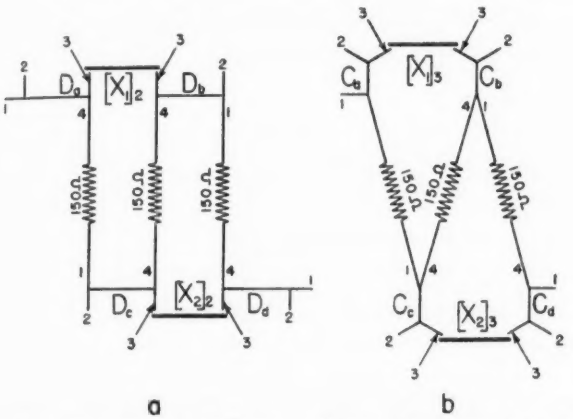

FIGURE 26.—*Two arrangements of linear resistances for determining the the effect of three different current distributions in each of the four nonlinear four-terminal blocks shown in figure 13.*

consideration of the arrangement shown in figure 26(a), and the third partial solution may be obtained from a consideration of the arrangement shown in figure 26 (b). Here D_a, D_b, D_c, and D_d represent the direct resistances and C_a, C_b, C_c, and C_d represent the cross resistances of the terminal blocks, *a*, *b*, *c*, and *d*.

Let

$[R_s]_2$=the four-terminal resistance of the entire combination with the series connection, excepting that part contributed by the cross resistances C_a, C_b, C_c, and C_d of the terminal blocks.

$[R_p]_2$=the four-terminal resistance of the entire combination with the parallel connection, excepting that part contributed by the cross resistances.

$[X_1]_2$ and $[X_2]_2$=the resistances of the paralleling connections, excepting that part contributed by the cross resistances.

$[R_s]_3$=the four-terminal resistance of the entire combination with the series connection, excepting that part contributed by the direct resistances, D_a, D_b, D_c, and D_d.

$[R_p]_3$=the four-terminal resistance of the entire combination with the parallel connection, excepting that part contributed by the direct resistances.

$[X_1]_3$ and $[X_2]_3$=the resistances of the paralleling connections, excepting that part contributed by the direct resistances.

L_1 and L_2=the four-terminal resistances of the paralleling connections.

R_s=the four-terminal resistance of the entire combination with the series connection, and

R_p=the four-terminal resistance of the entire combination with the parallel connection.

Referring to figure 26(a), it is readily seen that

$$[R_s]_2 = 450 + D_a + D_b + D_c + D_d, \tag{83}$$

$$[R_p]_2 = 50 + (9D_a + 4\,[X_1]_2 + D_b + D_c + 4\,[X_2]_2 + 9D_d)/9, \tag{84}$$

$$[X_1]_2 = L_1 - D_a - D_b, \text{ and} \tag{85}$$

$$[X_2]_2 = L_2 - D_c - D_d. \tag{86}$$

From eq 83 it follows that

$$[R_s]_2 = 450\,[1 + (D_a + D_b + D_c + D_d)/450], \tag{87}$$

and from eq 84, 85, and 86 it follows that

$$[R_p]_2 = 50\,[1 + (4L_1 + 4L_2 + 5D_a - 3D_b - 3D_c + 5D_d)/450]. \tag{88}$$

Referring to figure 26 (b), it is easily seen that

$$[R_s]_3 = 450, \tag{89}$$

$$[R_p]_3 = 50 + (4\,[X_1]_3 + 4\,[X_2]_3 - 6C_a - 6C_d)/9 \tag{90}$$

$$[X_1]_3 = L_1 + C_a + C_b, \text{ and} \tag{91}$$

$$[X_2]_3 = L_2 + C_c + C_d. \tag{92}$$

From eq 89, 90, 91, and 92 it follows that

$$[R_s]_3 = 450, \text{ and} \tag{93}$$

$$[R_p]_3 = 50 + (4L_1 + 4L_2 - 2C_a + 4C_b + 4C_c - 2C_d)/9. \tag{94}$$

Transferring the additive terms of eq 88 which contain the direct resistances to eq 94 gives

$$R_p = 50\,[1 + (4L_1 + 4L_2 + 5D_a - 2C_a - 3D_b + 4C_b - 3D_c + 4C_c + 5\,D_d - 2C_d)/450]. \tag{96}$$

Transferring the additive terms of eq 87 which contain the direct resistances to eq 93 gives

$$R_s = 450\,[1 + (D_a + D_b + D_c + D_d)/450]. \tag{95}$$

From eq 95 and 96 it follows

$$R_s/R_p = 9[1 - 4(L_1 + L_2 + D_a - C_a/2 - D_b + C_b - D_c + C_c + D_d - C_d/2)/450], \tag{97}$$

which is eq 57 in the appendix on terminals and contacts.

It is readily seen that the resistance R_1, of terminal block a is higher by $2C_a/3$ and the resistance R_1, of terminal block d is higher by $2C_d/3$ with the parallel connection than with the series connection. In fact, this is the reason why C_a and C_d appear in eq 90.

Obviously the effects of the different current distributions in each of the four-terminal terminal blocks depend on their four-terminal resistances. Furthermore, by design or adjustment, these four-terminal resistances may be made exceedingly small. As an illustration of a design having exceedingly small four-terminal resistances consider an equilateral tetrahedron, using the apices as the terminals. Also, by design, it is possible to make the four-terminal resistances definite to almost any extent desired. In cases in which the four-terminal resistances are sufficiently small to be neglected, and are equally definite, a nonlinear four-terminal conductor may be considered as equivalent to four linear conductors connected in star.

In the solution of network problems, it is frequently possible to select for the independent variable any one of two or more quantities which are dependent on each other. Making different selections leads to different forms. Then, by making approximations, still other forms are obtained of the solution. As an illustration of exact expressions for the current in the galvanometer branch of an unbalanced bridge consider eq 76, 78, 79, and 81. The second of these involves the drop in potential across the bridge with the galvanometer branch open, the third the drop in potential across the bridge with the galvanometer branch

closed, the fourth the current through the bridge, while the first, that is the usual classical solution, involves the electromotive force in the battery branch. The second of these involves the resistance of the battery branch in a rather complicated way, as does also the classical solution. Therefore, if only these four were available it is conceivable that cases might arise in which it would be more convenient to use either the third or the fourth rather than the second or the classical form of solution. In many cases approximate solutions may be obtained in forms much more convenient to use than any of the forms of exact solutions, yet giving all the accuracy required.

APPENDIX 6. OHM'S LAW

The work of Ohm contributed materially to our concepts of current, electromotive force, and potential difference; originated our concepts of resistance and resistivity; gave the generally used relation between current, electromotive force, and resistance; and led to the formulas used in expressing the resistances of combinations of conductors in terms of their individual resistances and to various laws and theorems pertaining to the distribution of current in systems of conductors. However, more than 50 years before Ohm published his more important papers, Cavendish very probably had concepts of the quantities now called potential difference, current, resistance, and of the property now called resistivity. Furthermore, from experiments with electrolytic conductors he reached the conclusion that the current is proportional to the first power of the potential difference.

During 1825, 1826, and 1827, Ohm published a number of papers in which he described measurements of the electric conductance of metal wires, drew some general conclusions from his experimental data, and made an analysis of the electric circuit. These papers led to those general ideas and relations which taken collectively may be considered to be Ohm's law. From the standpoint of resistance comparisons, the most important of these general ideas is that the resistance of a conductor is independent of the current in it, while the more important of these relations are the equations used in expressing the resistances of combinations of conductors in terms of the resistances of the individual conductors.

The various conclusions reached by Ohm were not accepted unless or until verified by others. However, by 1843, when Wheatstone published an important paper, they were more or less generally accepted. Yet the feeling persisted that more accurate measurements would show Ohm's law to be merely an approximation. In the early 1879's it was thought that definite departures from the law had been found. Accordingly, the British Association appointed a committee to investigate and report on the matter. Maxwell was the chairman of this committee, and presumably he devised the methods of test, but the measurements were made by Chrystal. A conclusion reached from an analysis of the experimental data obtained in the tests and reported by Maxwell in 1876 was that "If a conductor of iron, platinum or German silver one square centimeter in cross section has a resistance of one ohm for infinitely small currents its resistance when acted upon by an electromotive force of one volt (provided the temperature is kept the same) is not altered by so much as $1/10^{12}$ part." These tests have usually been considered as proof that Ohm's law is exact, at least to the extent likely to be significant in electrical measurements. Nevertheless, the subject is deserving of some discussion.

An expression for the potential drop between the terminals of a standard resistor (in which there is a current) that presumably is more exact than that generally used is

$$e = RI + x + y + z, \tag{98}$$

where

e = the potential drop between the potential leads of the standard resistor,
R = the direct resistance of the standard resistor,
I = the component of the current under the control of the operator and which enters and leaves the standard resistor through the current terminals,
$x = Ri$, where i is the current not under the control of the operator,
y = the component of the potential difference having its origin in the standard resistor and potential leads and which is not caused by the current I, and
z = the component of the potential drop in the standard resistor and potential leads caused by changes in the magnetic field.

Letting E=that component of the potential drop caused by the current, I, it follows from eq 98 that

$$E=RI. \tag{99}$$

Pertinent questions concerning the application of these and other equations, based on Ohm's law, to standard resistors and resistance coils under normal conditions of use are:

1. To what extent is R independent of I?
2. To what extent is eq 99 reliable?
3. To what extent are the formulas used in expressing the resistance of combinations of conductors in terms of the resistances of the individual conductors reliable?
4. To what extent can E be separated from e?

Question 1.—A precision method of determining the extent to which R is independent of I consists in the use of a bridge (either Wheatstone or Thomson) and observing the changes in balance with changes in the magnitude of the test current and the changes in balance with the time following the establishment or reversal of the test current. The procedure followed in establishing balances must be such as to give a separation of E and e. Questions 2 and 3 are not involved.

Obviously only relative effects of the test current upon the resistances of the conductors constituting the arms of the bridge can be determined in this way. Therefore, a quantitative conclusion as to the effect upon some particular standard resistor can be reached only after a number of observations have been made, and this conclusion must involve some assumption such as that the effect for a selected group of standard resistors is, on the average, zero, except to the extent that causes are found and the effects of these causes are taken into consideration.

In the normal use of standard resistors the test current causes the resistance material to assume a temperature above that of the surrounding medium. This results in what may be considered as the normal change in the resistance of the conductor corresponding to the difference in temperature. In addition, the difference in temperature between the resistance material and its mechanical support may cause strains in the conductor, and if so, these strains result in a further change in the resistance. These changes in resistance are at least approximately proportional to the square of the current and reach their full magnitude only after the current has been maintained a sufficient time for the difference in temperature to become substantially constant.[6] These will be referred to as the primary effects of the current upon the resistance. A procedure for determining and applying a correction for the primary effects of the test current has been considered under the heading "Load Coefficients."

In addition, the current affects the resistance, in some cases at any rate, in other ways. When a number of standard resistors are substituted one after another in the X arm of the bridge and balances are established by the procedure outlined above, it not infrequently happens that the precision attainable for these balances is less with some of the standards than with others, although all may have low temperature coefficients and equal facilities for dissipating the heat developed by the test current. In cases in which the precision attainable in establishing a balance of the bridge is well below the average, it not infrequently happens that the resistance of the resistor under investigation is low immediately following a reversal of the test current and then increases more or less gradually attaining a practically constant value in a time ranging from a few seconds to a minute or more. It happens less frequently that the resistance changes in an irregular way and does not come to a practically constant value.

A comparison of the performance characteristics with the details of construction, properties of the materials used in the construction, insulation resistance, etc., frequently give a definite clue as to the cause of the changes in resistance. In some cases it appears that following a change in the current the Peltier effect, especially at junctions between copper and the resistance material, causes a local increase and a local decrease in the temperature, which results in a thermoelectromotive force. This electromotive force, after it reaches a constant value, is proportional to the current and reverses sign following a reversal of the direction of the current. In other cases, it appears that an appreciable part of the current passes through the insulation, and that this part decreases for a time (relative to

[6] It is conceivable (in cases in which the conductor is not rigidly supported) that the forces resulting from electromagnetic action might produce strains of sufficient magnitude to cause perceptible changes in the resistance. If so, these changes would appear almost instantly following a change in the magnitude of the current.

the total current) following the establishment or reversal of the test current. In still other cases it appears that there is a gradual mechanical yielding of the insulating material under the stresses caused by the temperature difference between the resistance material and its mechanical support. In such cases the effect of the test current upon the resistance may be larger at the end of 15 seconds than at the end of a longer time.

Changes in resistance resulting from the test current, whether or not the way in which they are brought about is known, and provided they are not proportional to the square of the current, will be referred to as the secondary effects of the current upon the resistance. There are many possible causes of secondary effects, so it is not possible in all cases to obtain as complete information concerning them as might be desired.

However, correlation of the secondary effects with details of construction of many resistors makes it possible to predict with some degree of certainty their magnitudes in others in which they are too small to be observed. For relatively few resistance coils and standard resistors under normal conditions of use is it probable that the secondary effects of the current upon the resistance amount to less than 1 in 100 million. On the other hand, for relatively few standard resistors having resistances in the range from 0.0001 to 10,000 ohms, if of the precision type and of good quality, are the secondary effects of the normally used test currents in excess of 1 in 100,000.

Question 2.—It appears that the first precise measurements having a direct bearing on the relation between a change in current from one steady value to another steady value and the resulting change in potential drop were made by F. A. Wolff of this Bureau. At a meeting of the American Association for the Advancement of Science in the summer of 1899 he presented a paper entitled "Experimental Test of the Accuracy of Ohm's Law." Unfortunately, no further record of this investigation has been found either in the form of a published paper or unpublished notes or data. The circuit used was essentially that shown in figure 27. Here x_1, x_2, x_3, and x_4 represent the four arms of a balanced Wheatstone bridge. This bridge constitutes the X arm of a second Wheatstone bridge, the other arms of which are designated Y, A, and B. This bridge is also balanced. Dr. Wolff refers to the bridge whose arms are designated x_1, x_2, x_3, and x_4 as the auxiliary bridge, the other bridge as the main bridge, and the circuit as a bridge within a bridge.

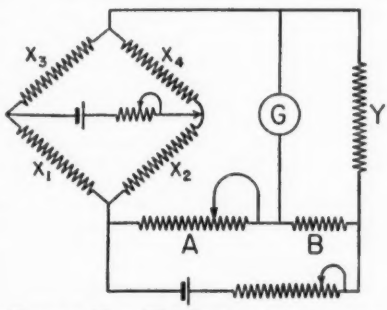

FIGURE 27.—*Wolff's circuit for testing the accuracy of Ohm's law.*

This method was tried about 1920 [105], when it gave promise of reasonably satisfactory results, though some difficulties were encountered. These difficulties were, for the most part, obviated by replacing the battery of the auxiliary bridge by a source of alternating current and omitting a provision for balancing the auxiliary bridge. Standard resistors having nominal values of 10 ohms were used as the x_1, x_2, x_3, and x_4 arms of the auxiliary bridge and the Y arm of the main bridge. The direct current in the Y arm was adjusted to a value somewhat larger than that regularly used in testing 10-ohm standard resistors and the alternating current in the supply branch of the auxiliary bridge was adjusted to a root mean square value equal to approximately twice the value of the direct current in the Y arm of the main bridge.

Since x_1, x_2, x_3, and x_4 were approximately equal in resistance very little of the alternating current passes through the A, B, and Y arms, or through the battery or galvanometer branch of the main bridge. Presumably, therefore, the alternating current had no appreciable effect except in x_1, x_2, x_3, and x_4, the arms of the auxiliary bridge. Following the usual procedure in balancing the main bridge, the precision of the balances was about 1 part in 10 million. On opening the connection to the source of the alternating current, the balance was changed by an amount not perceptibly different from that corresponding to the change in resistance of the series parallel combination of x_1, x_2, x_3, and x_4 resulting from the reduced power dissipation.

A conclusion to be drawn from this experiment, for standard resistors such as were used and currents having values over a limited range, including values normally used in testing such standards, is that part of the potential drop caused by the current is equal to the product of the resistance and current to an extent at least somewhat beyond that to which the resistance may be considered as independent of the current. It appears that if apparatus were designed especially for the purpose and the most suitable resistance wire obtainable were used, the method might be expected to yield a precision of a few parts in 100 million.

Question 3.—The test made by Chrystal, using "the first method which occurred to the Committee," might have given directly an answer as to the reliability of one of the formulas used in expressing the resistance of a combination of conductors in terms of the resistances of the individual conductors. In this method five resistance coils of equal resistance are compared with each other, four in a 2-and-2 series-parallel combination. According to Ohm's law, the resistance of each coil is the same as the resistance of the other four in the series-parallel combination, and on the average this would be true, even if there were slight differences in the resistances of the five coils. However, the quality of the resistors used by Chrystal was not sufficiently good to permit of measurements to what now would be considered a high accuracy.

An insignificant modification of the method consists in the comparison of the resistances of four standard resistors (of nominally equal resistance) with each other and with the four in the series-parallel combination, by alternate substitution in the same arm of a Wheatstone bridge. In a test made about 1920 [105], using 10-ohm standard resistors of good quality, this method gave reasonably satisfactory results, when corrections were applied to take into account the effects of the resistances of terminals and contacts, but not sufficiently satisfactory to be considered as a definite answer as to the reliability of one of the formulas used in expressing the resistance of a combination of conductors in terms of their individual resistances. However, there seems to be no direct or indirect experimental evidence (such as that given in the answer to question 2) not in conformity with the supposition that the resistances of combinations of conductors, as expressed in terms of their individual resistances, are reliable to the extent that the resistances of the individual conductors are definite. But to realize this accuracy it may be necessary to take into account not only the resistances of connectors but also the effects of the different current distributions in terminal blocks or the different current distributions through or over the surface of the insulation.

Question 4.—The extent to which the potential drop E resulting from the current I and the actual potential drop e can be separated may be judged by deliberately introducing into a bridge circuit a disturbing electromotive force and noting the resulting change in the balance and precision of the balance. In cases in which the disturbing electromotive force amounts to only a few microvolts and is reasonably constant or is only two or three orders higher and alternating at frequencies of power distribution systems (or higher) neither the balance nor the precision of the balance is affected by amounts corresponding to as much as 0.01 microvolt. This question has been discussed more fully in appendix 2, Thermoelectromotive Forces.

This brief discussion of Ohm's law has been limited mainly to questions arising in precision resistance comparisons. It represents an attempt to coordinate one of several classes of information obtained more or less incidentally in the design, construction, and testing of standard resistors, bridges, potentiometers, etc.

APPENDIX 7. UNITS OF RESISTANCE

A comprehensive discussion of units of resistance is not pertinent to the subject of this paper. However, some reference should be made here to the unit now known as the NBS International Ohm and the relations between this unit and some of the other units now being used or whose use is contemplated.

The International Conference at London in 1908 established an International Committee on Electrical Units and Standards to formulate a plan for and direct such work as may be necessary in connection with maintenance of standards, fixing of values—intercomparison of standards and to complete the work of the conference. This International Committee met in Washington in the spring of 1910, and one of its decisions was "to accept (for the present) as the International ohm the mean of the units of the Physikalisch-Technische Reichsanstalt and the National Physical Laboratory" which had been realized in these laboratories from the so-called mercury ohm.

Standard resistors from Germany, France, and England were compared with standard resistors of the United States from time to time during a period of about 5 weeks. Values were then assigned to all the standard resistors used in the intercomparisons in terms of the new unit. In assigning the new values, it was assumed that the mean value of one of the standard resistors from the National Physical Laboratory during the time of the intercomparisons was the same as the value previously assigned in the National Physical Laboratory, and that the mean of the values of the resistances of the two standard resistors from the Physikalisch-Technische Reichsanstalt during the time of the intercomparion was the same as the mean of the values previously assigned in the Physikalisch-Technische Reichsanstalt. Equal weights were given to the previously assigned value in the National Physical Laboratory and the mean of the previously assigned values in the Physikalisch-Technische Reichsanstalt.

Following the assignment of new values to the standard resistors of this Bureau used in the intercomparison, they were intercompared either directly or indirectly with many other standard resistors of this Bureau. From time to time since then, selected groups (usually of 20 or more) 1-ohm standard resistors have been intercompared, and on the basis of relative changes and previous good performances a group of 10 standards has been selected from the larger group. In assigning new values to all the standard resistors of the larger group and to others, it has been assumed that the mean resistance of the group of 10 had not changed since the previous similar intercomparison. This in general is the procedure by which the unit established in 1910 has been maintained in this Bureau.

From 1910 to 1929 only standard resistors of the sealed type developed by Rosa [76] were included in the groups of 10. A total of 20 different standards have been included in the groups of 10, while 3 of these 20 have been included in every group of 10. From 1932 to 1938 only standards of the smaller double-walled type [89] were included in the groups of 10. More recently, only standards of the larger double-walled type constructed by Thomas in 1933 have been included in the groups of 10.

The unit established in 1910 and maintained in this Bureau in the way described above has been referred to by different names. The name now being used is the NBS International Ohm.

Obviously, a unit of resistance maintained in this way changes with time to the extent of the proportional decrease (or increase) of the resistances of the standard resistors used in its maintenance. Therefore, it is not to be presumed that the NBS International Ohm has remained constant during the time since its establishment. To obtain information on this point various investigations have been made in this Bureau. Most of those since 1927 have been made by Thomas. In addition, groups of standard resistors have been sent abroad for measurements of their resistances in other national laboratories, and standards of other national laboratories have been measured in this Bureau. More recently these international intercomparisons have been made at the International Bureau of Weights and Measures. Comparisons of the values found for the resistances of the same standard resistors in different national laboratories have given the relations between the units of resistance of the different countries. From information as to the manner in which the units of the different countries were established and have been maintained, the results of the international intercomparisons and the investigations made in this Bureau, it may be concluded that the probable change in the NBS International Ohm during the 30 years since its establishment has been less than 30 parts in 1 million. Since the standard resistors now being used in this Bureau in the maintenance of the units are fully one order better than those formerly used for this purpose, it is reasonable to presume that the present rate of change of the NBS International Ohm is less than 1 part in 1 million per year.

The relation between the NBS International Ohm, the mean international ohm, and the units of resistance of other countries is given in table 1, which contains data taken from the 1933, 1935, and 1937 reports of the International Committee of Weights and Measures (Comité International des Poids et Mesures, Procés-Verbaux des Séances) and Comptes Rendus, page 24, volume 209, 1939.

TABLE 1.—*Units of resistance of various countries, as determined from comparisons of standards at the International Bureau of Weights and Measures* [1]

[Expressed as departures in microhms from the Mean International Ohm]

Unit of—	Third comparison, November 1933	Fourth comparison, March 1935	Fifth comparison, December 1936	Sixth comparison, February 1939
Germany	+10.6	+9.8	+6.6	+9.1
United States	−6.4	−5.5	−3.7	−3.2
France	+7.3	+69.5		
		0.0	+0.9	+3.7
Great Britain	−5.2	−3.6	−3.9	−6.5
Japan	−8.3	−11.2	−10.0	−14.4
Russia	+9.5	+10.6		
		0.0	−0.4	+0.8

[1] In this table a plus sign signifies that the unit of the country was larger than the Mean International Ohm.

The Mean International Ohm was defined as the mean of the units of electrical resistance, in 1935, of Germany, United States, Great Britian, Japan, and Russia. After this mean was obtained, it was taken by France as the new French unit and by Russia as the new Russian unit. The data given in the table are based on the assumption that units of resistance are being maintained with equal fidelity in each of the six countries.

The unit of the United States is the unit of resistance referred to above as the NBS International Ohm. Therefore, in the early part of 1939,

1 NBS International Ohm = 0.999 997 Mean International Ohm.

From measurements made in this Bureau by Wenner, Thomas, Cooter, and Kotter, the results of which were reported to the International Bureau of Weights and Measures in December 1938,

1 NBS International Ohm = 1.000 485 absolute ohm.

From measurements made in this Bureau by Curtis, Moon, and Sparks, the results of which were reported to the International Bureau of Weights and Measures in December 1938,

1 NBS International Ohm = 1.000 484 absolute ohm. In each case the uncertainties in the measurements were estimated to be 2 parts in 100,000.

In 1937 the International Committee on Weights and Measures adopted provisionally the relation

1 Mean International Ohm = 1.000 48 absolute ohm.

From this relation and the data given in table 1, it follows that provisionally

1 NBS International Ohm = 1.000 477 absolute ohm

as of February 1939.

APPENDIX 8. REFERENCES

This list of references is by no means comprehensive, but an effort has been made to include most of the little known publications containing contributions of importance at the time. In cases in which original publications are not readily available, references are made to later publications or to text and handbooks.

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The letters included in brackets at the end of each reference indicate the subjects discussed. The key to these letters is given under the heading "Subjects" p. 293.

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(b) SUBJECTS

The numbers included in brackets at the end of each subject refer to publications in which the subject is discussed. The key to these numbers is given under the heading "Authors."

[A] Adjustable resistance elements. [63, 120].
[B] Balancing of resistances of connectors. [45, 74, 82, 101, 115, 121].
[C] Current distribution in a linear network. [29, 30, 49, 100, 108].
[D] Differential galvanometer method. [3, 31, 33, 34, 39, 40, 50, 51, 52, 54, 65, 82, 86].
[E] Four-terminal conductors. [25, 35, 58, 80, 101].
[F] Galvanometers. [6, 14, 15, 28, 37, 38, 40, 41, 43, 44, 54, 56, 59, 61, 62, 65, 75, 86, 100, 101, 104, 116, 119, 122, 123].
[G] Galvanometer shunts. [2, 4, 17, 44, 65, 95, 107, 120].
[H] Measurement of resistances of connectors. [44, 46, 55, 82, 111, 121].
[I] Ohm's law. [5, 13, 57, 66, 67, 69, 70, 105, 106, 109, 113].
[J] Potentiometer method. [40, 60, 82, 86].
[K] Sensitivity of bridges. [9, 11, 19, 20, 21, 23, 25, 26, 28, 29, 32, 39, 40, 42, 51, 58, 65, 68, 71, 78, 79, 82, 83, 84, 85, 86, 94, 100, 101, 125].
[L] Series-parallel build up. [24, 72, 82, 83, 84].
[M] Shunting of standard resistors. [55, 73, 81, 84].
[N] Star-polygon transformations. [48, 53, 77].
[O] Thermoelectromotive forces. [7, 8, 22, 27, 29, 120].
[P] Thomson bridge. [1, 19, 21, 25, 36, 45, 46, 54, 55, 65, 74, 81, 83, 84, 94, 101, 121].
[Q] Shielding. [16, 47].
[R] Standard resistors. [76, 89, 90, 91, 92, 93, 98, 102, 110].
[S] Various methods and their modifications. [10, 18, 19, 25, 54, 58, 64, 65, 82, 83, 84, 85, 87, 99, 101, 103, 114, 115].
[T] Wagner branch. [16, 29, 47, 96, 97].
[U] Wheatstone bridge. [12, 29, 47, 82, 86, 98, 112, 117, 118].

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